Linear Regression

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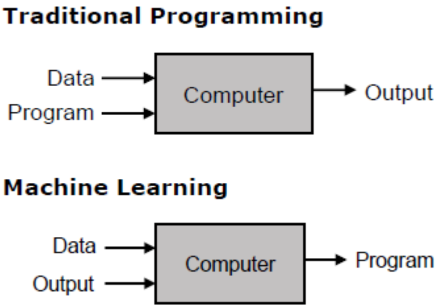
Talk Overview

1. ML inroduction
2. ML Terminologies
3. ML Applications Examples
4. Linear Regression

Outline

So What is Machine Learning?

- Automating automation
- Getting computers to program themselves
- Writing software is the bottleneck
- Let the data do the work instead!



Machine Learning Problem Types

- **Based on Type of Data**
    - Supervised, Unsupervised, Semi supervised, Reinforcement Learning
- **Based on Type of Output**
    - Regression, Classification
- **Based on Type of Model**
    - Generative, Discriminative

We will focus on: Supervised → {Regression, Classification} → Discriminative models

Types of Learning based on Type of Data

- Supervised learning
    - Training data includes desired outputs.
    - Trying to learn a relation between input data and the output
- Unsupervised learning
    - Training data does not include desired outputs.
    - Trying to “understand” the data.
- Semi supervised learning
    - Training data includes a few desired outputs.
- Reinforcement learning
    - Rewards from sequence of actions.

Types of Learning based on Type of Output

Regression

A regression model predicts continuous values.

For example:

- What is the value of a house in California?
- What is the probability that a user will click on this ad?

Classification

A classification model predicts discrete values.

For example:

- Is a given email message spam or not spam?
- Is this an image of a dog, a cat, or a hamster?

Types of Learning based on Type of Model

Generative Model

A Generative nodel explicitly learns the actual distribution of each class.

Discriminative Model

A Discriminative model learns the decision boundary between the classes.

Generative Models

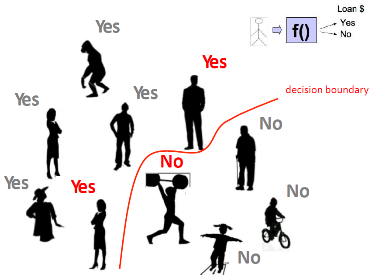
- Naïve Bayes
- Hidden Markov Models
- Bayesian networks
- Markov random fields

Discriminative Models

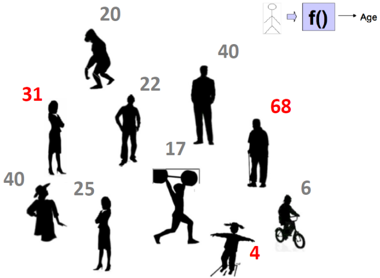
- Logistic regression
- SVMs
- Traditional neural networks
- Nearest neighbor
- Conditional Random Fields (CRF)

Regression vs Classification

Classification (supervised learning)



Regression (supervised learning)



Outline

ML Basic Terminologies

Many terminologies associated with ML. Will cover them in the following slides.

- Labels
- Features
- Examples
- Models

Label

- A label is the thing that we are predicting in classification or regression task. **Example:** Male, Female
- The label could also be the future price of wheat, the kind of animal shown in a picture, the meaning of an audio clip, or just about anything.
- Usually denoted with the variable y.

Features (1/2)

- A feature is an input variable (a.k.a attribute).
- A simple machine learning project might use a single feature, while a more sophisticated machine learning project could use millions of features.
- Usually denoted as:

$$x_1, x_2, \ldots, x_N$$

In the spam detector example, the features could include the following:

- words in the email text
- sender's address
- time of day the email was sent
- ...

Features (2/2)

Generally three types of attributes:

- **Categorical:** red, blue, brown, yellow
- **Ordinal:** poor, satisfactory, good, excellent
- **Numeric:** 3.14, 6E23, 0,

Features (2/2)

Generally three types of attributes:

- **Categorical:** red, blue, brown, yellow
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Categorical

- No natural ordering to categories
- Categories usually encoded as numbers

Ordinal

- There is a natural ordering to categories
- Encoded as numbers to preserve ordering

Numeric

- Integers or real numbers
- meaningful to add, mul tiply, compute

Notethat

The process of generating this features for our machine learning problem is called feature engineering.

Data samples (Examples)

Data sample / Example is a particular instance of data, **x**. (Note That. **x** is a vector of features)

We break examples into two categories:

- Labeled examples: (Used for prediction)
- Unlabeled examples: (Used for inference/testing)

Example

housingMedianAge (feature)	totalRooms (feature)	totalBedrooms (feature)	medianHouseValue (label)
15	5612	1283	66900
19	7650	1901	80100
17	720	174	85700
14	1501	337	73400
20	1454	326	65500

Model

A **model** defines a relationship between features and label.

Two phases of a model's life:

- **Training** means creating or learning the model. You show the model labeled examples and enable the model to gradually learn the relationships between features and label.
- **Testing/Inference** means applying the trained model to unlabeled examples. You use the trained model to make useful predictions (y').

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ML Application 1: Credit Approval

- **Numeric Features:**
  - loan amount (e. g. $1000)
  - Income (e. g. $65000)
- **Ordinal Features:**
  - savings: {none, <100, 100..500, 500..1000, >1000}
  - employed: {unemployed, <1yr, 1..4yrs, 4..7yrs, >7yrs}
- **Categorical Features:**
  - purpose: {car, appliance, repairs, education, business}
  - personal: {single, married, divorced, separated}
- **Labels (Categorical):**
  - Approve credit application
  - Disapprove credit application

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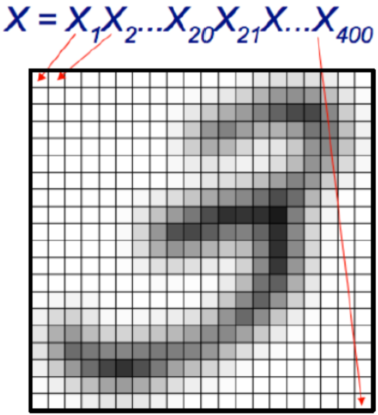
Easy feature engineering process.

ML Application 2: Handwritten Digits Recognation

Represent each pixel as a separate attribute either Categorical **OR** Ordinal:

- **Categorical Features:**
  - (white) or (black) based on a threshold
- **Ordinal Features:**
  - Degree of "blackness" of a pixel
- **Labels (Categorical):** {0,1,2,3,4,5,6,7,8,9}



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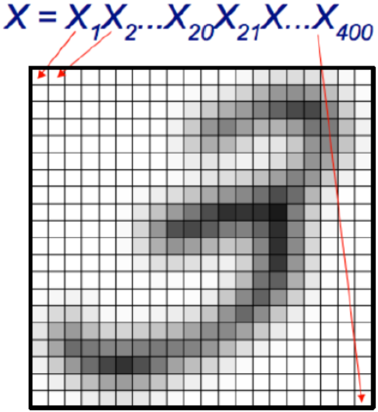
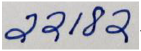


What if we are dealing with paper like this 

?

Isolate each digit, rescale, de-slant, ..

Hard feature engineering process.

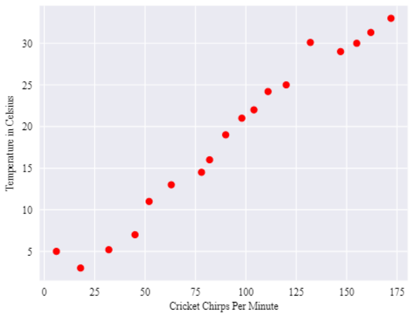
Outline

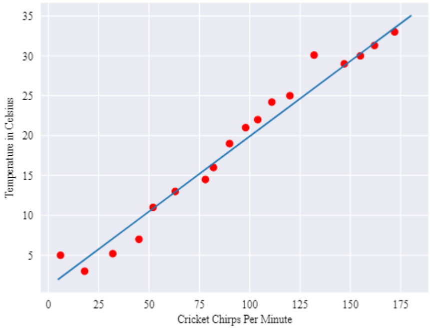
Linear Regression

Linear Regression

Linear regression attempts to model the relationship between two variables by fitting a linear equation to observed data

Example: Scientists found that crickets (an insect species) chirp more frequently on hotter days than on cooler days.





Linear Regression

A linear relationship

- True, the line doesn't pass through every dot.
- However, the line does clearly show the relationship between chirps and temperature.

$$y = mx + b$$

where:

- y: is the temperature in Celsius—the value we're trying to predict.
- m: is the slope of the line.
- x: is the number of chirps per minute—the value of our input feature.
- b: is the y-intercept.

Linear Regression

In machine learning, we'll write the equation for a model slightly differently:

$$y' = w_1 x_1 + w_0$$

where:

- y': is the predicted label (a desired output).
- w_1: is the weight of feature 1. Weight is the same concept as the "slope".
- x_1: is feature 1.
- w_0 or b: is the bias (the y-intercept).

Notethat

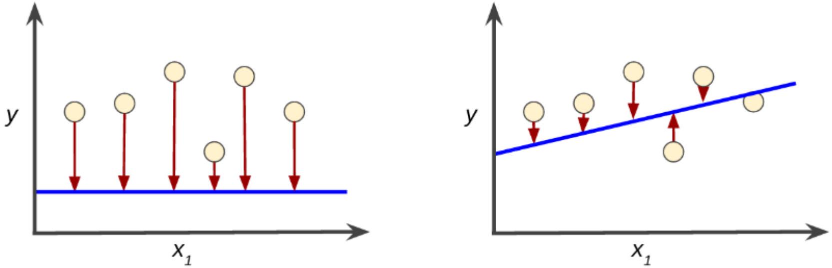
A model that relies on three features might look as follows:

$$y' = w_3 x_3 + w_2 x_2 + w_1 x_1 + w_0$$

Training and Loss

- **Training** a model simply means learning (determining) good values for all the weights and the bias from labeled examples.
- **Loss** is the penalty for a bad prediction.
    - Perfect prediction means the loss is zero
    - Bad model have large loss.
- Suppose we selected the following weights and biases.

Which of them have lower loss?



Squared loss

- The linear regression models use a popular loss function called squared loss.
- Also known as L_2.
- Is represented as follows:

$$[obsevation(x) - prediction(x)]^2$$
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Why squared loss?
Can we do absolute loss?

Mean square error (MSE)

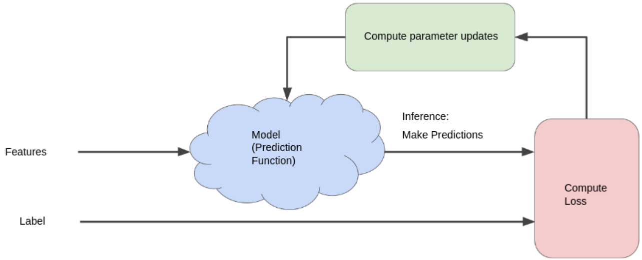
- Is the average squared loss per example over the whole dataset.

$$\text{MSE} = \frac{1}{N} \sum_{(x,y) \in D} (y - prediction(x))^2$$

- (x,y) is an example in which
  - y is the label
  - x is a feature
- prediction(x) is equal $y' = w_1 x + w_0$
- D is the dataset that contains all (x,y) pairs
- N is the number of samples in D

Reducing Loss

- **Training** is a feedback process that use the loss function to improve the model parameters.
- The **training** is an iterative process.



Two Questions

- What initial values should we set for w_1 and w_0?
- How to update w_1 and w_0?

Gradient Descent (1/3)

- Assume (for symplicity) we are only concerned with finding w_1.
- Assume we had the time and the computing resources to calculate the loss for all possible values of w_1.

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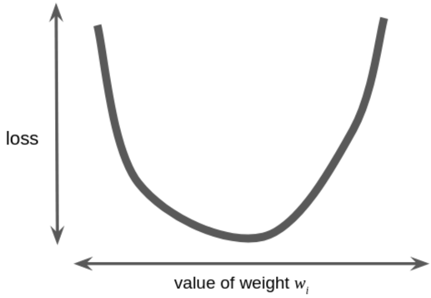
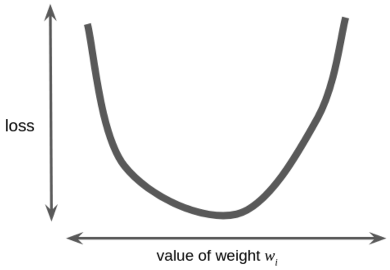
Regression problems yield convex loss vs. weight plots.



Gradient Descent (2/3)

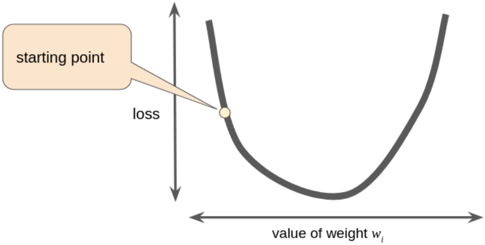
- Gradient descent enables you to find the optimal w without computing for all possible values.
- Gradient descent has the following steps
    1. Pick a random starting point for w
    2. Calculates the gradient of the loss curve at w.
    3. Update w
    4. go to 2, till convergence



Gradient Descent (3/3)

Note that a gradient is a vector, so it has both of the following characteristics:

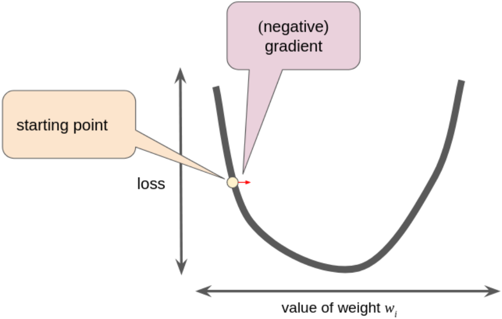
- Magnitude
- Direction



$$w_{new} = w_{old} - \eta * \frac{d \ \ loss}{dw}$$

Gradient Descent (3/3)

The gradient descent algorithm takes a step in the direction of the negative gradient



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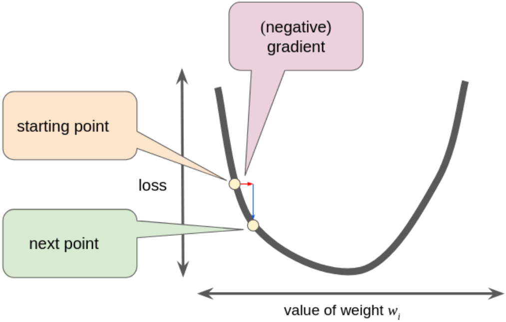
Gradient Descent (3/3)

the gradient descent algorithm **adds some fraction of the gradient's magnitude (Learning Rate η)** to the previous point



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Convergence Criteria

- For convex functions, optimum occurs when
  - $|\frac{d \ loss}{dw}| = 0$
- In practice, stop when
  - $|\frac{d \ loss}{dw}| \leq \epsilon$

Learning rate

- Gradient descent algorithms multiply the gradient by a scalar known as the learning rate (also sometimes called step size) .
- How can we choose the learning rate?

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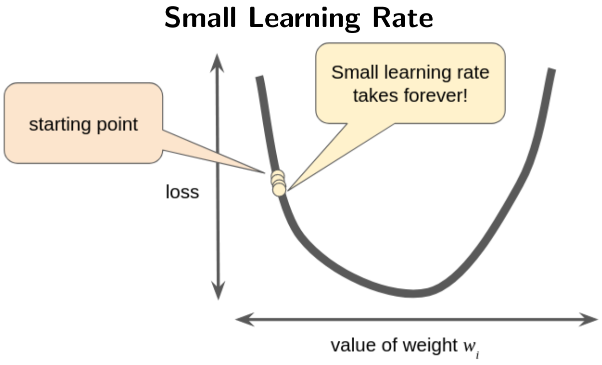

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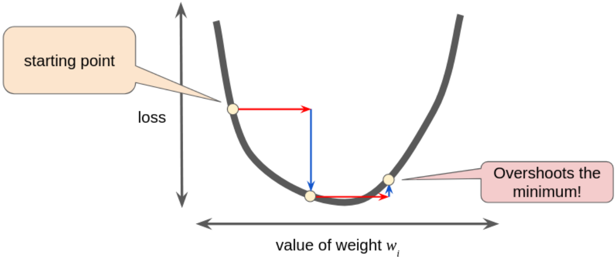
Large Learning Rate



Learning rate

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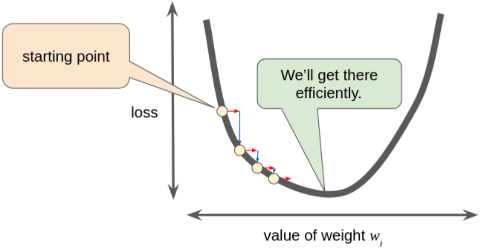
Optimal Learning Rate usually (0.01)



Generalization and Gradient

- For n features: $y' = \sum_{i=0}^{i=n} w_i x_i$
- Note w_0 is the bias (intercept), and $x_0 = 1$.
- vector representation y' = $\mathbf{w}^T\mathbf{x}$
- Loss = $\ell = (y - y')^2$
- Gradient derivation

$$\begin{aligned}\frac{d\ell}{dw_i} &= \frac{d\ell}{dy'}\frac{dy'}{dw_i} \\ &= [2(y - y') * x_i * (-1)]\end{aligned}$$

Types of Gradient Descents

- **Batch Gradient Descent:**
  - MSE loss assumes taking gradient for the total number of samples in the data set
  - Data sets often contain billions or even hundreds of billions of examples
  - Can take a very long time to compute.
- **Stochastic Gradient Descent (SGD):**
  - Uses only a single example (a batch size of 1) per iteration.
  - Very noisy.
- **Mini-Batch Gradient Descent:**
  - Compromise between full-batch iteration and SGD
  - Typically a batch of size between 10 and 1,000 examples, chosen at random.

Questions