ECEN 377: Engineering Applications of AI

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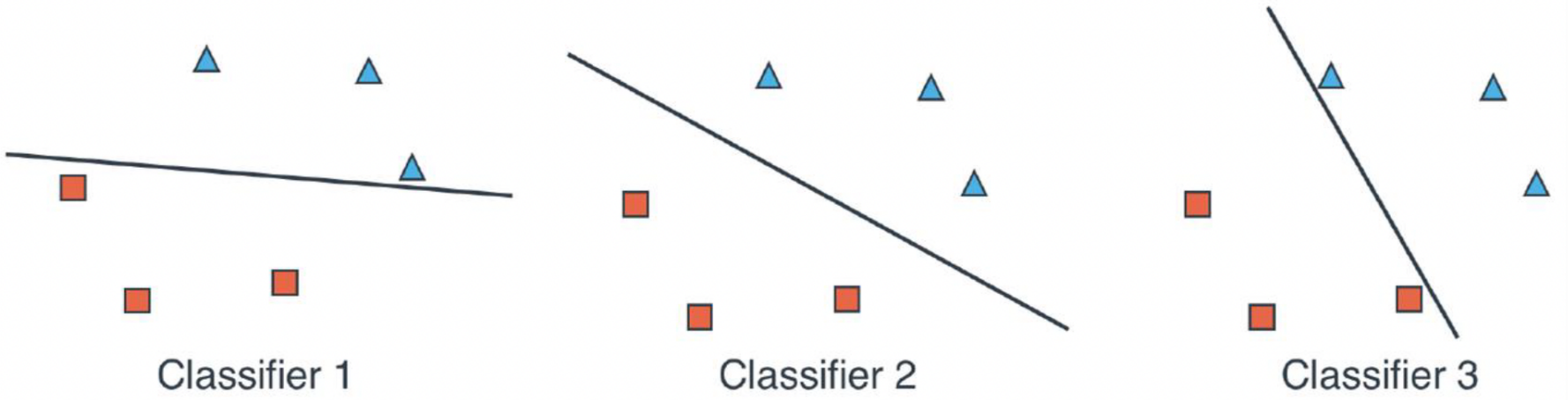
Outline

Introduction to SVM

Which of the following is the best classifier? and **why?**



Introduction to SVM

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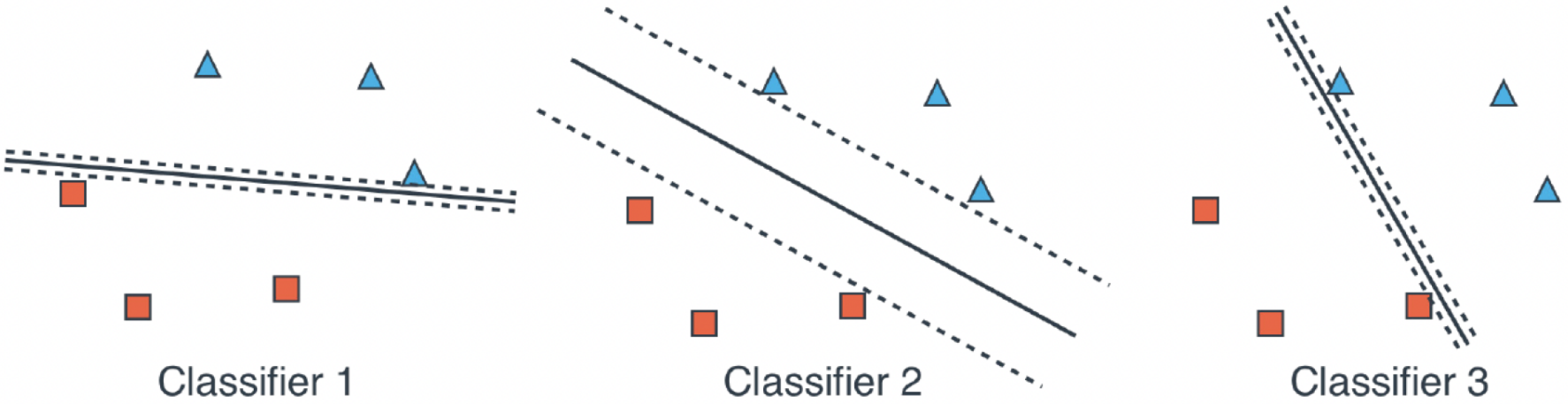


Classifier 2 is better because it has a **larger margin** between the classes. **Margin** is the distance between the decision boundary and the closest data points. (Support Vectors)

Outline

SVM Loss Function

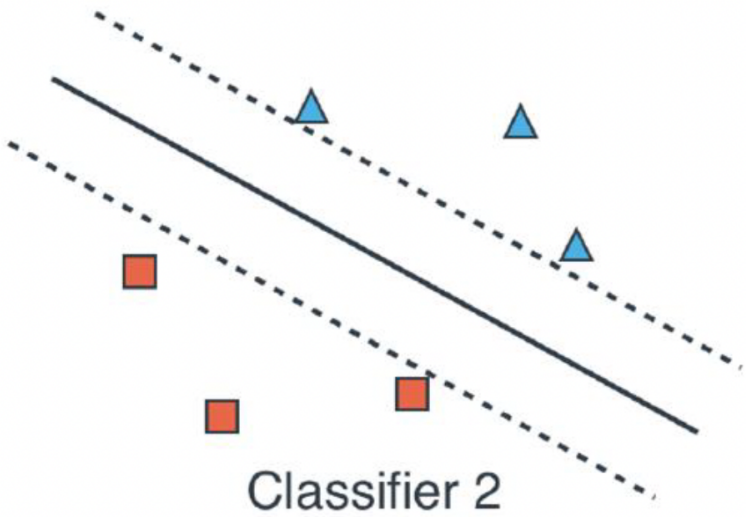
How we should penalize misclassified data points and maximize the margin?

Loss Function =

Classification Loss

+

Distance Loss



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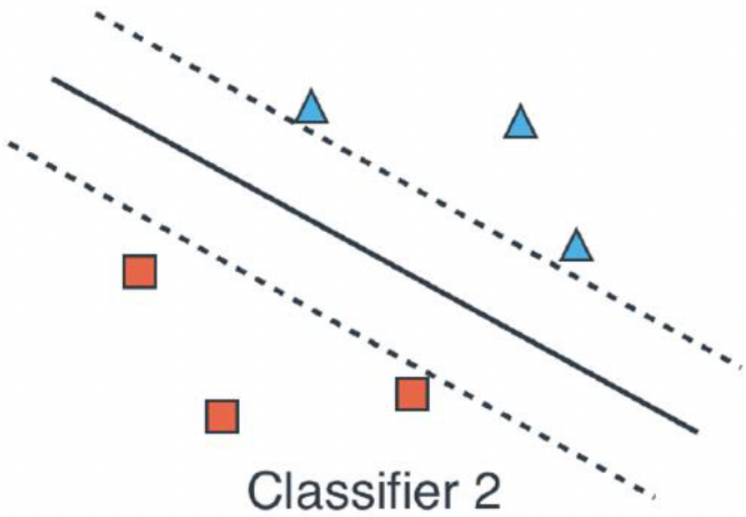


Assume we aim to train a linear SVM on 2D data points (x_1, x_2).

Outline

Classification Loss Function

In SVM, Instead of using only the decision boundary, we use two parallel **imaginary lines** shifted by bias.

Example:

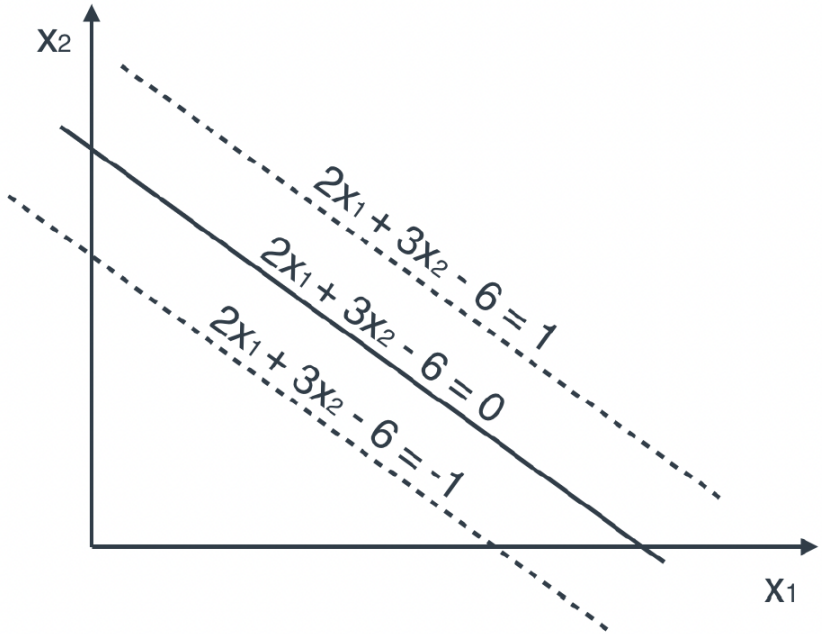
Decision Boundary: $2x_1 + 3x_2 - 6 = 0$

Imaginary Line (+ve):

$2x_1 + 3x_2 - 6 = 1$

Imaginary Line (-ve):

$2x_1 + 3x_2 - 6 = -1$



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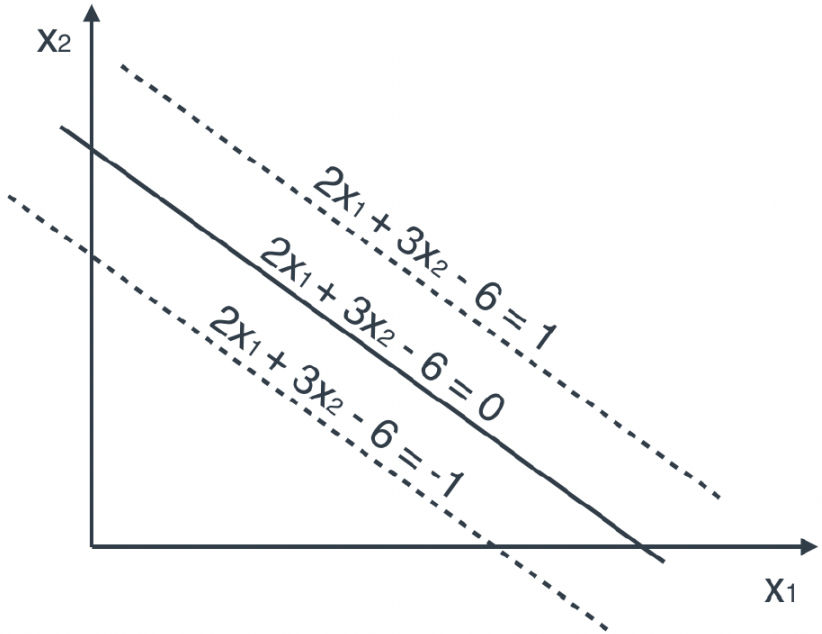

Note: Even correctly classified data points within the margin should be penalized! **How?**

Classification Loss

Requirement for Classification Loss:

- Correct classification above the +ve margin has a loss value of zero.

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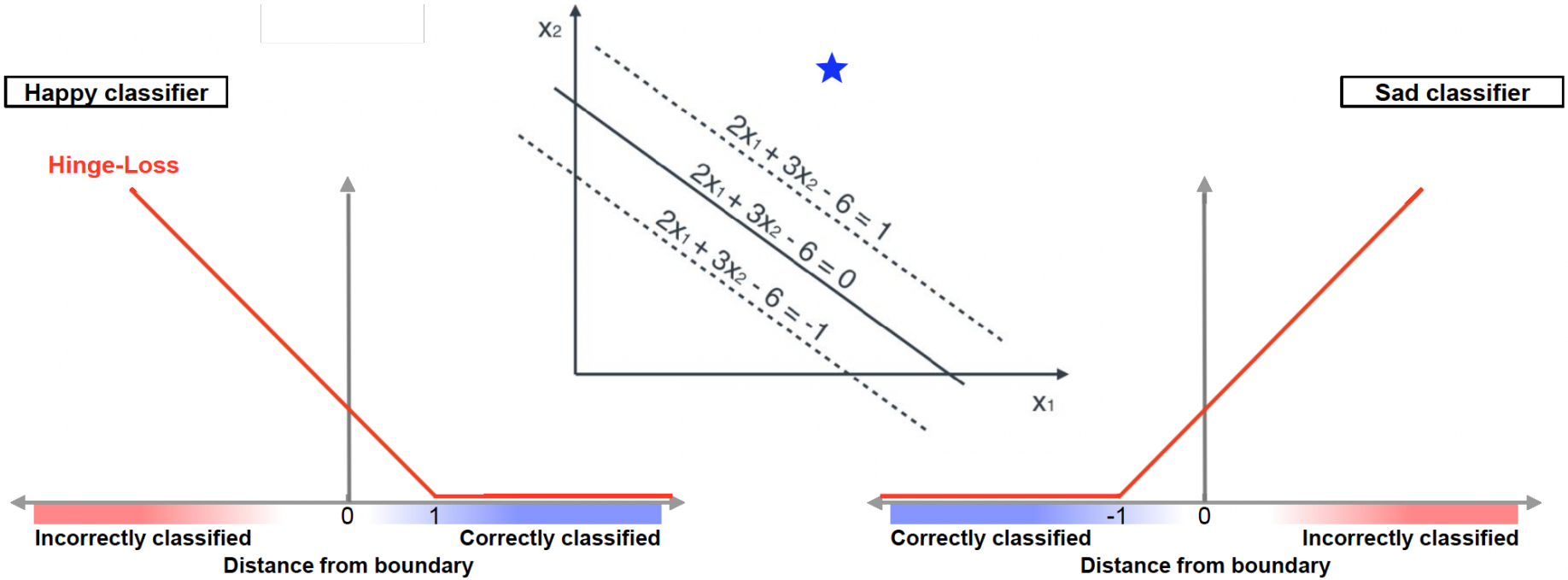
- Misclassified data points below the -ve margin have a loss value greater than zero.
- Correctly classified data points within the margin have a loss value greater than zero.

Hinge Loss (Classification Loss)

The Hinge Loss function is defined as:

$$L_{\text{classification}}(y, \hat{y}) = \max(0, 1 - y \cdot y')$$

where y is the true label (either $+1$ or -1) and y' is the predicted label.



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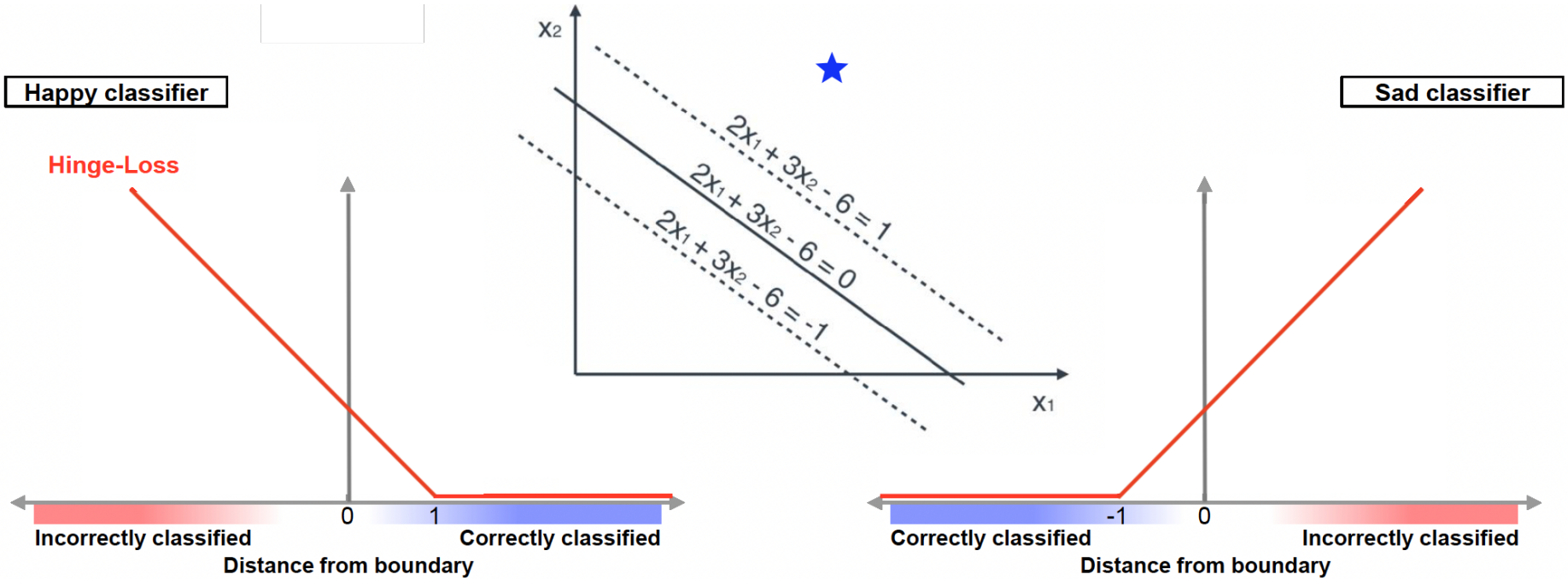

What is the loss for the blue data point if it is correctly classified? Which area is the loss in?

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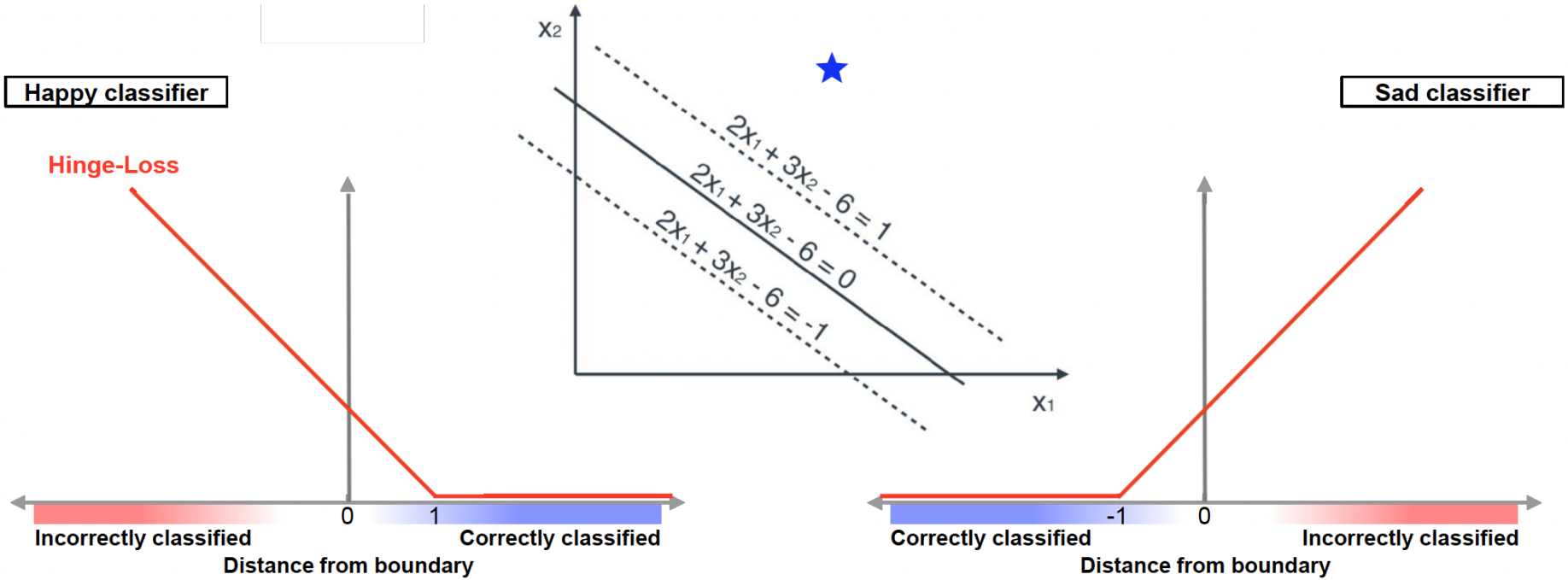

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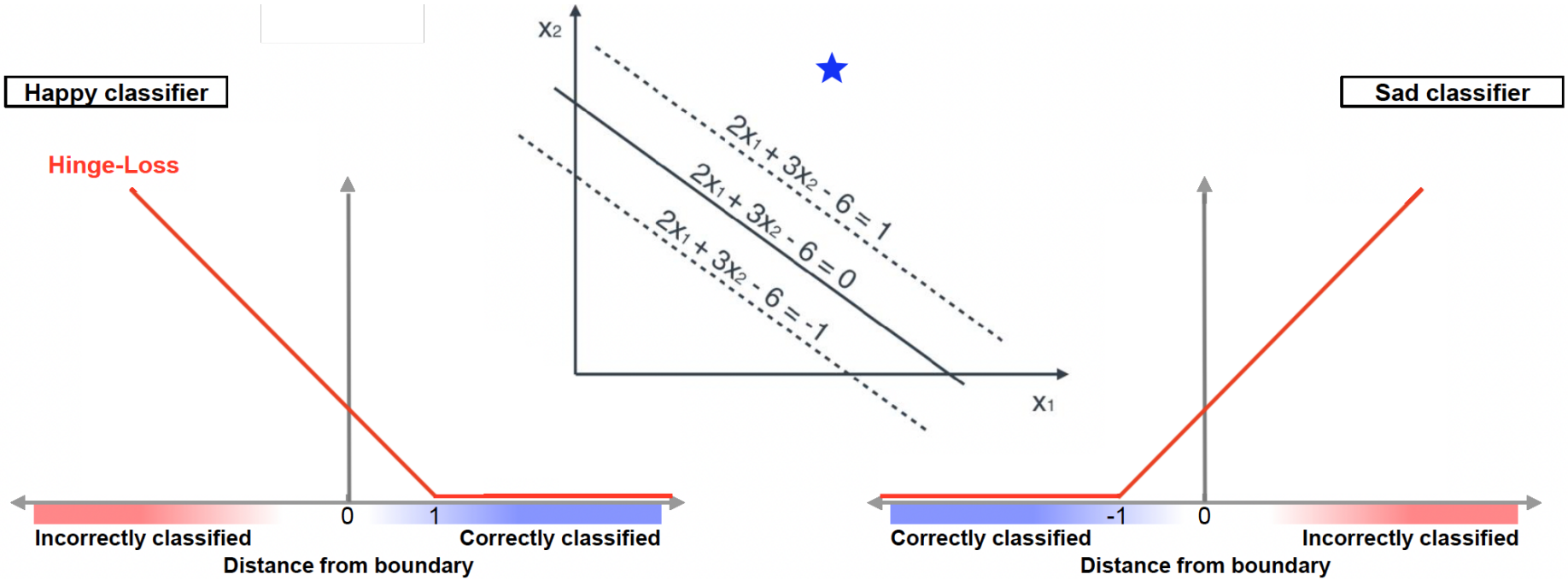

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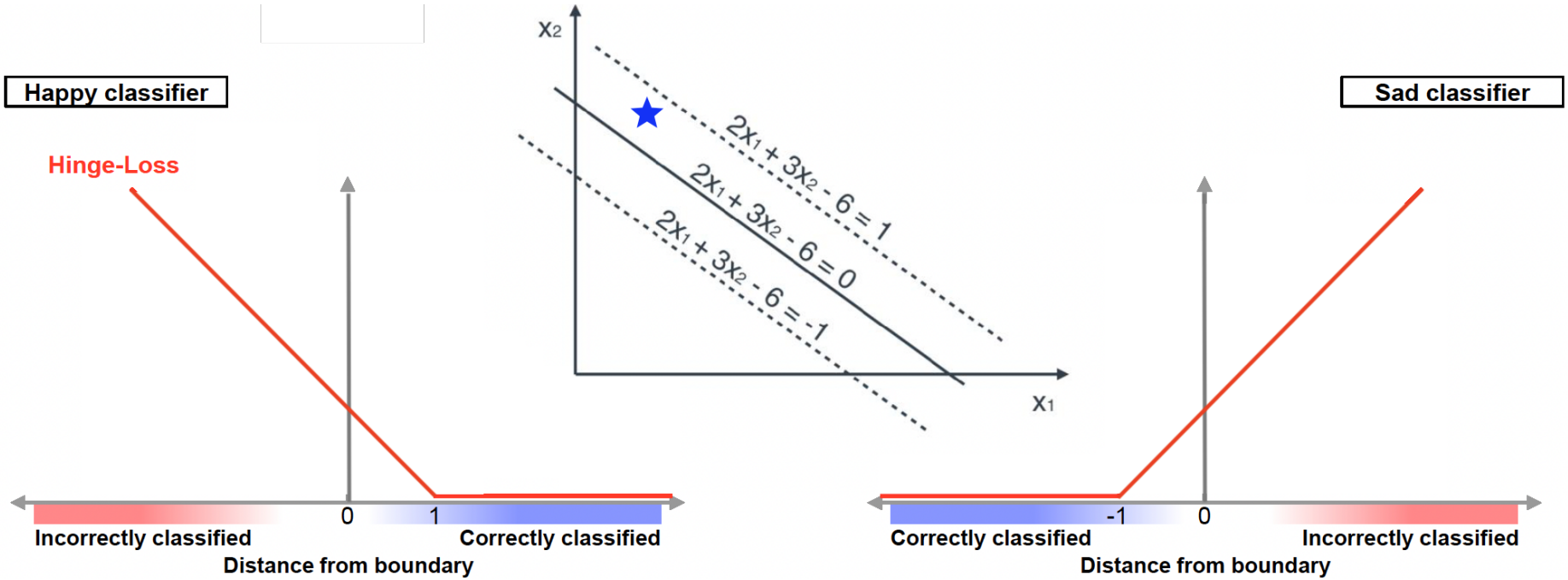

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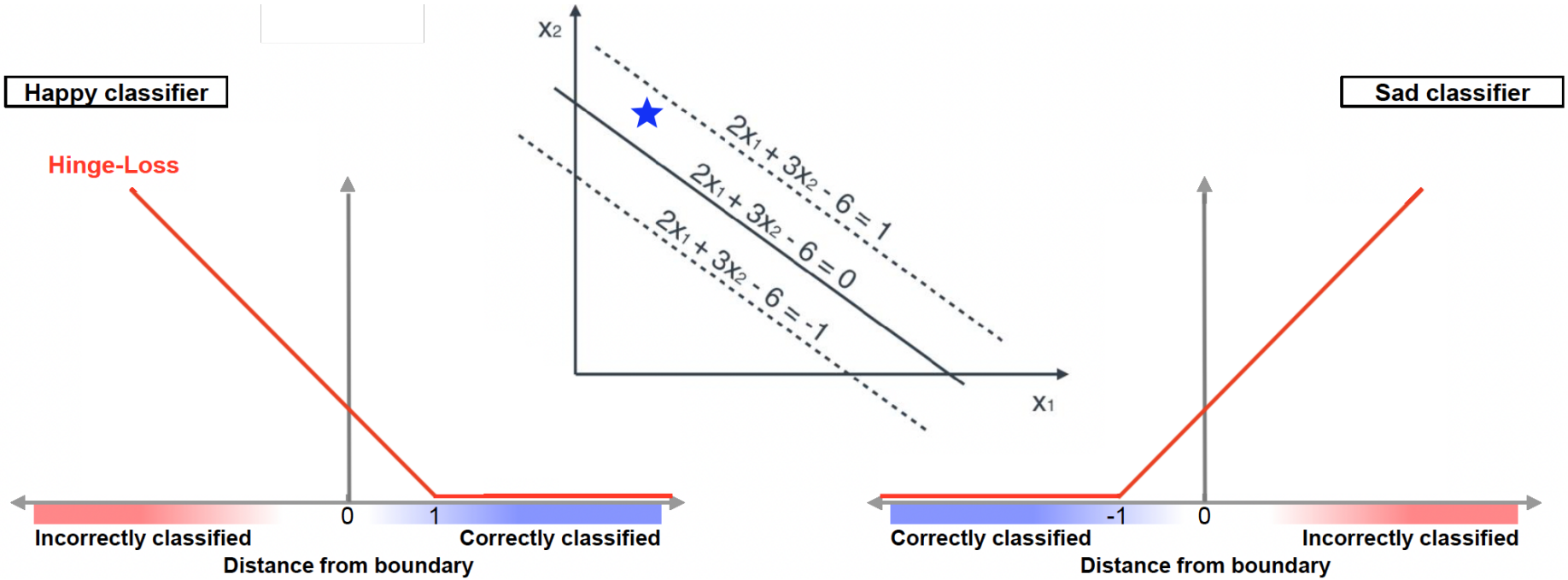

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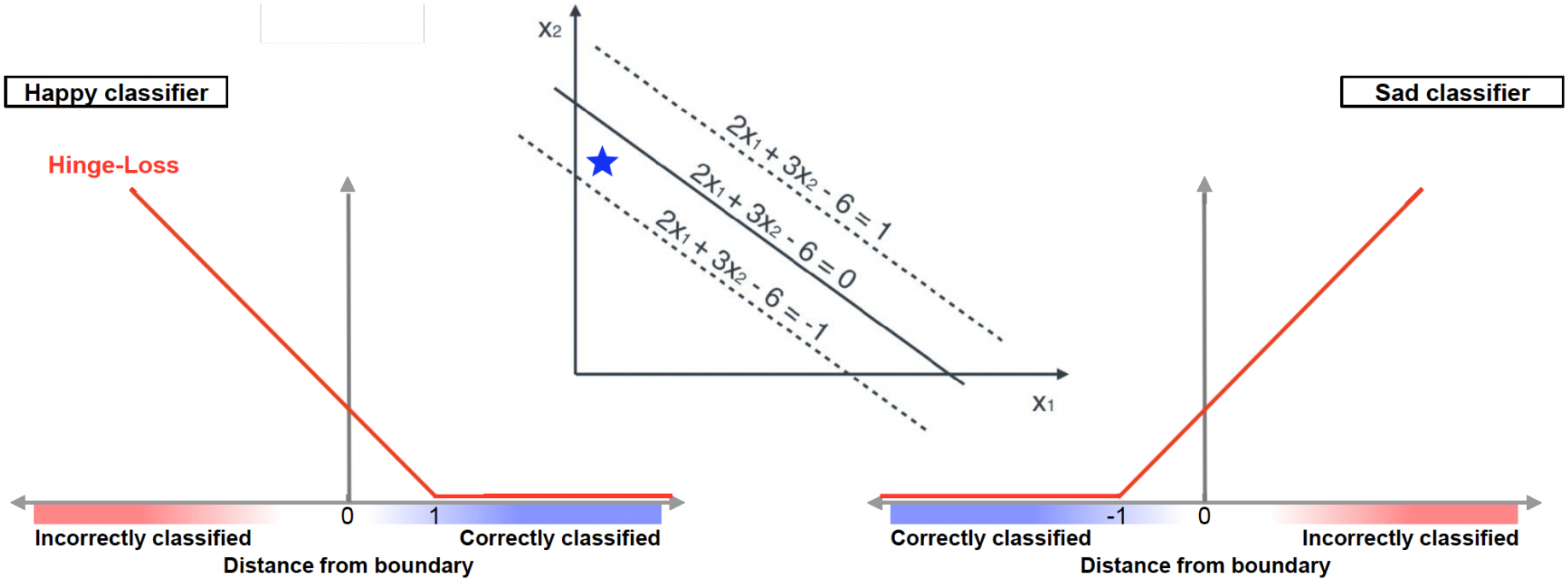

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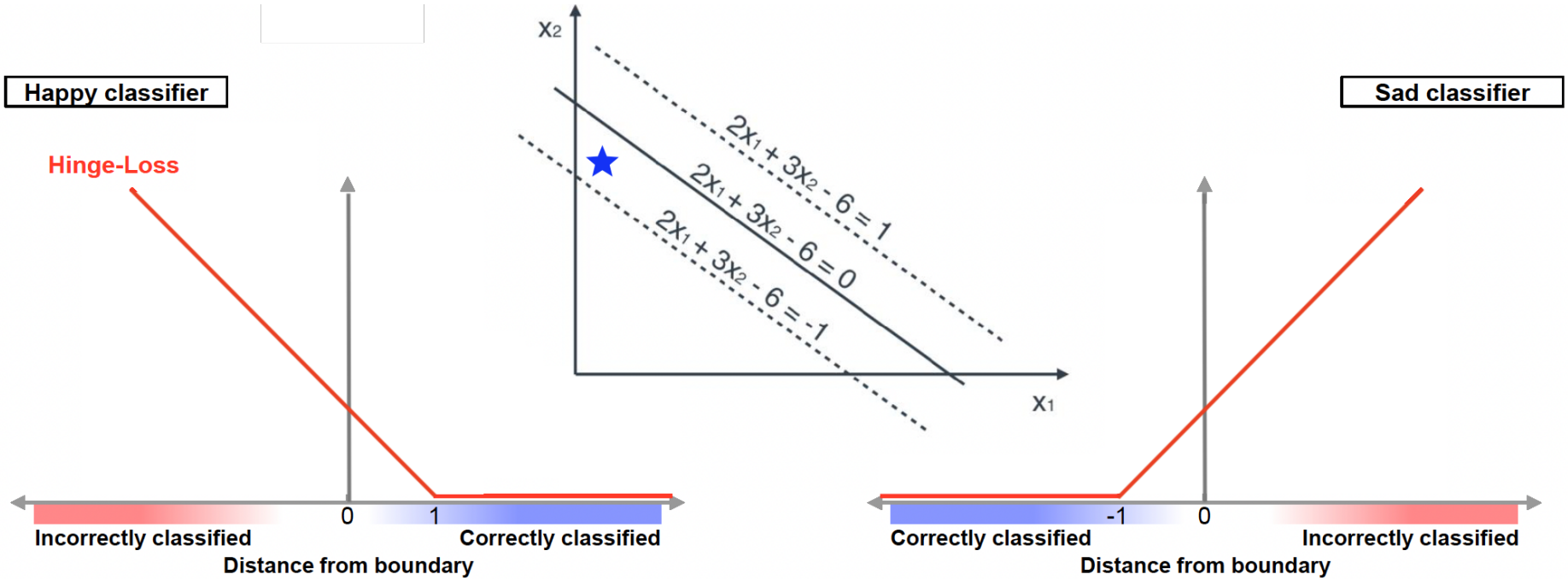

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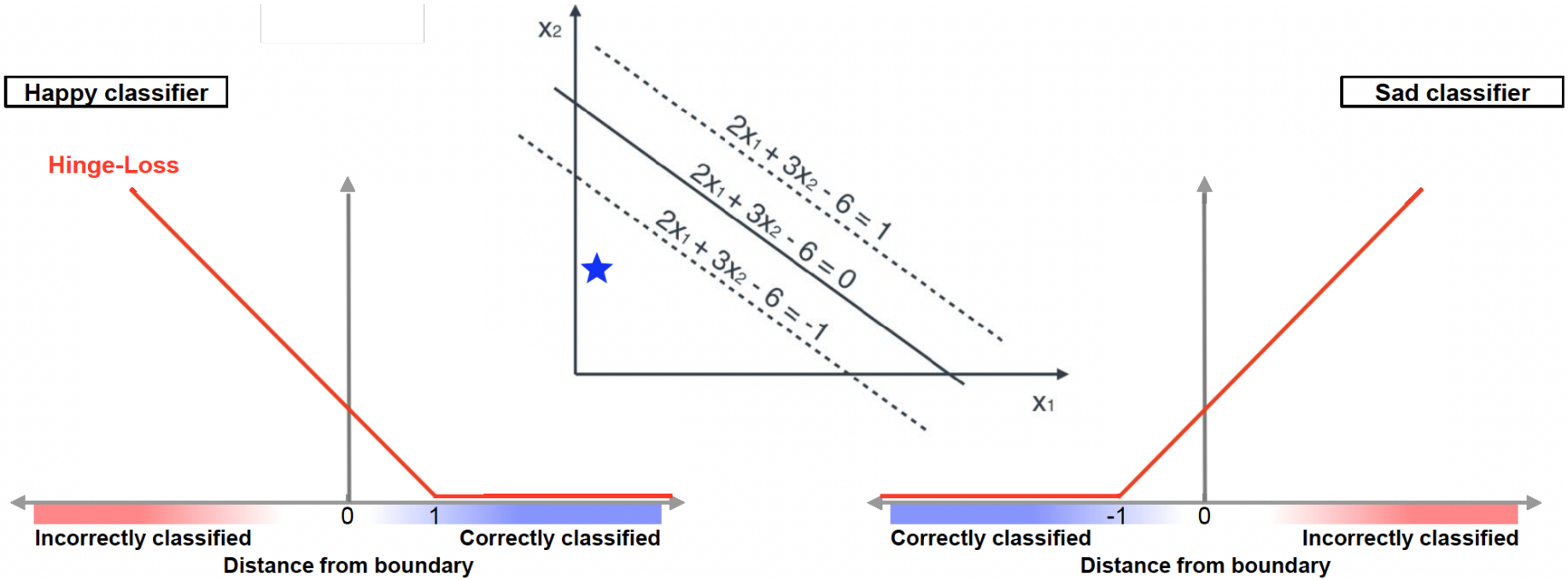

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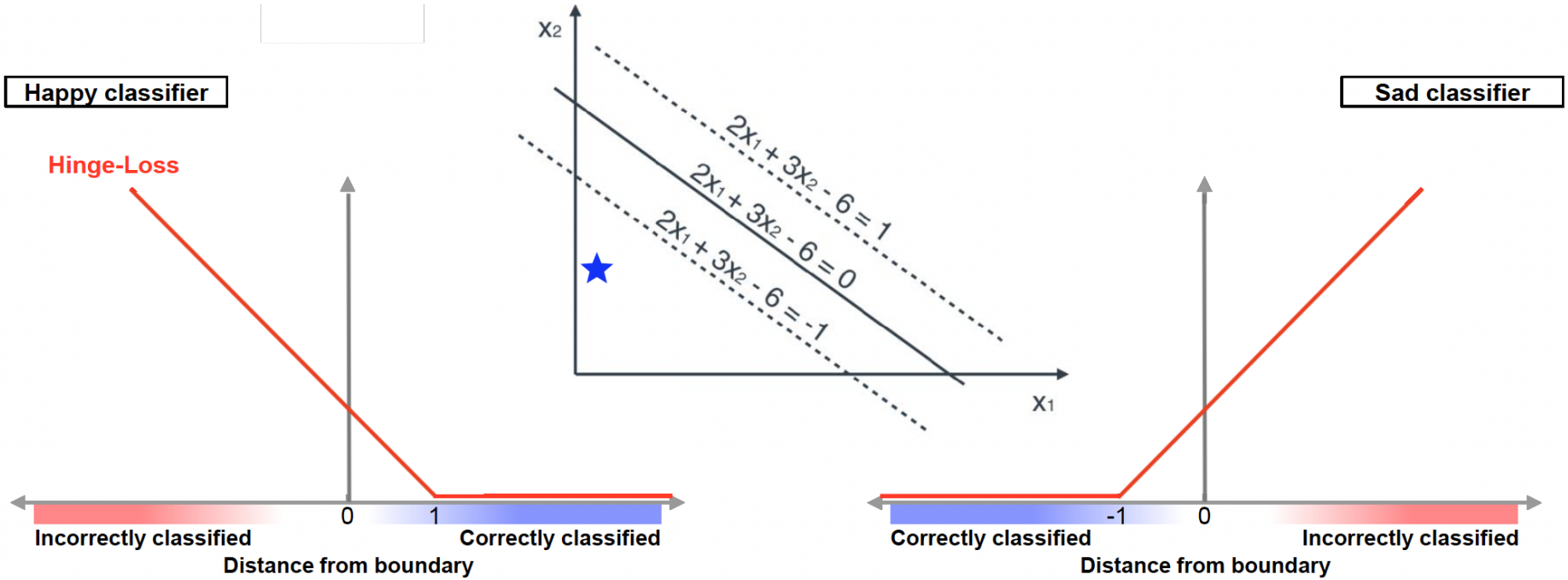

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Outline

Distance Loss Function (Large Margin)

What is the distance between two parallel straight lines?

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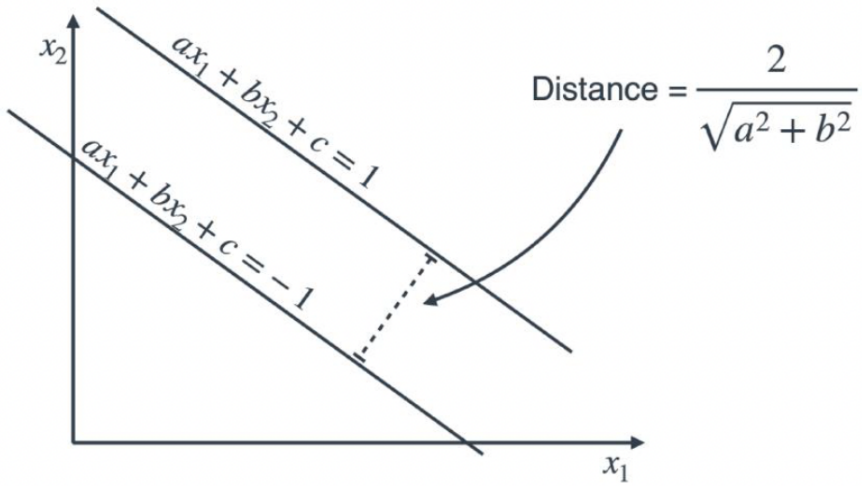
More general form:

$$\frac{2}{\|w\|} = \frac{2}{\sqrt{w_1^2 + w_2^2 + w_3^2 + \cdots + w_n^2}}$$



Note:

- The larger the weight vector, the smaller the (margin) Bad classifier.
- The smaller the weight vector, the larger the (margin) Good classifier.

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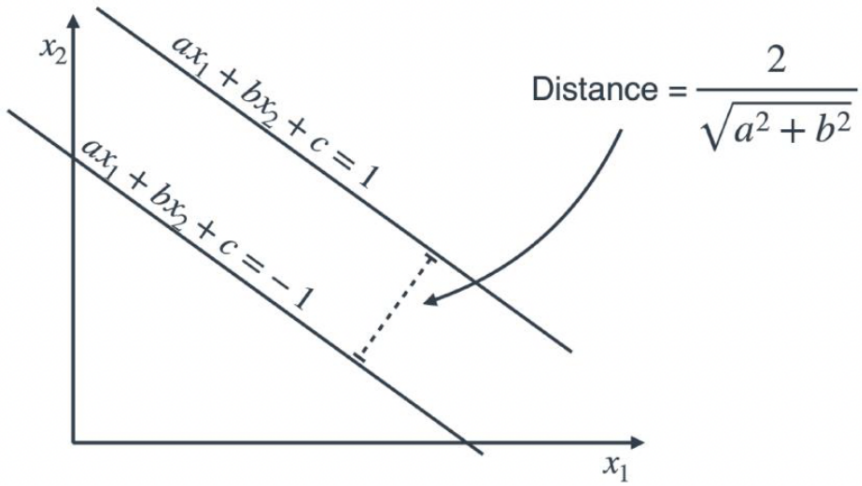

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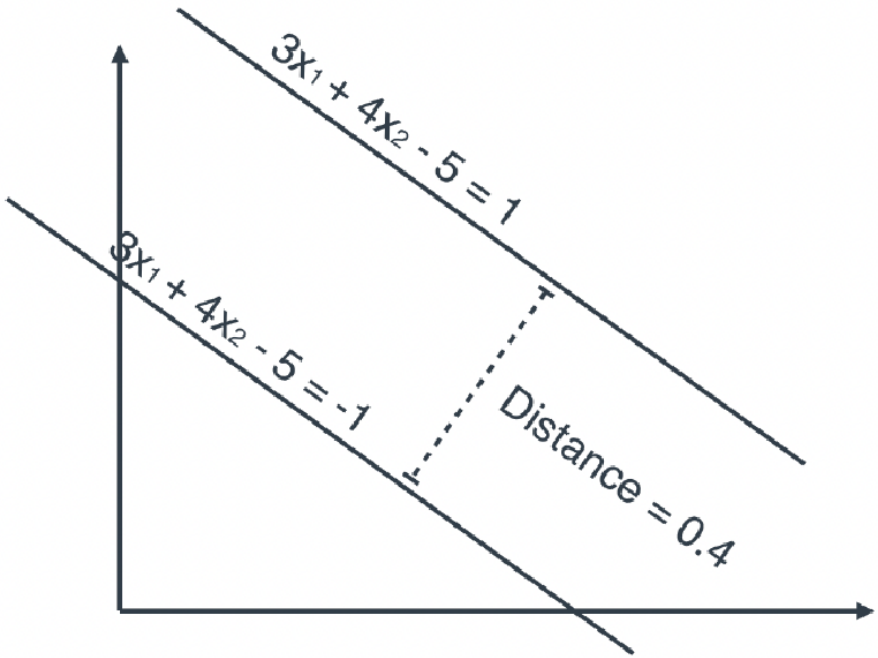
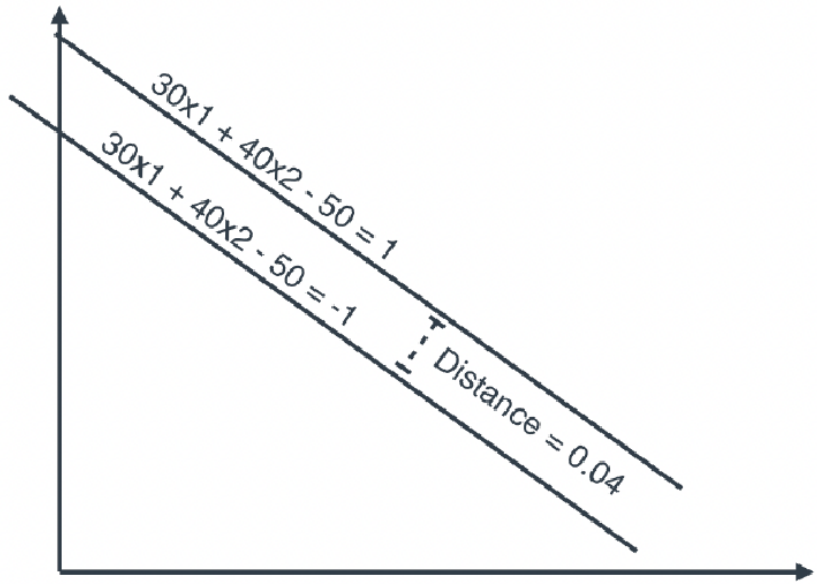
There for maximizing the margin is equivalent to minimizing the weight vector!

Distance Loss Function (Large Margin)



Error function = 25
Good classifier



Error function = 2500
Bad classifier

Outline

Overall Loss Function

The overall loss function is the sum of the classification loss and the distance loss:

$$L(y, y') = \underbrace{L_{\text{classification}}(y, y')}_{\text{Hinge Loss}} + \underbrace{\frac{1}{2}\|w\|^2}_{\text{Regularization} \equiv \text{Large Margin}}$$

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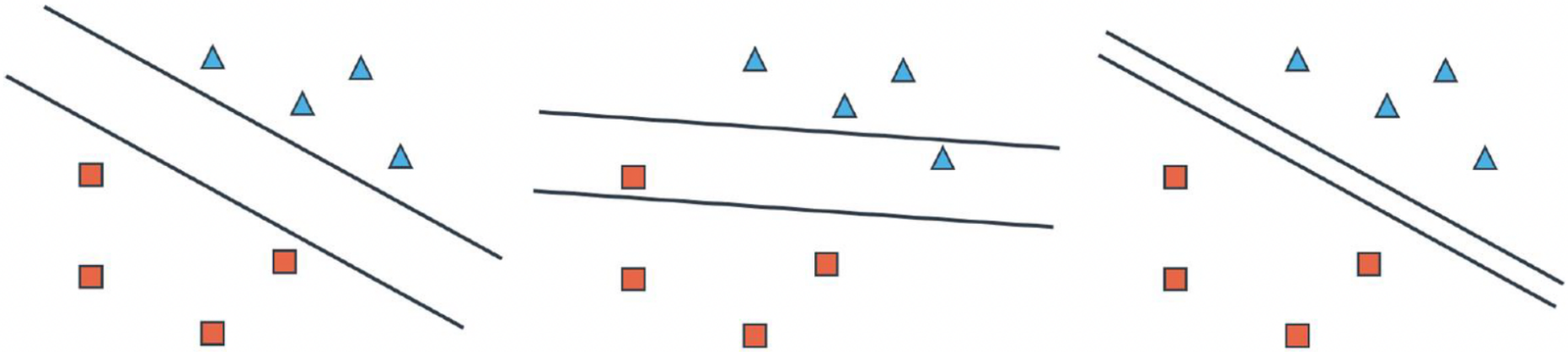
Overall Loss Function



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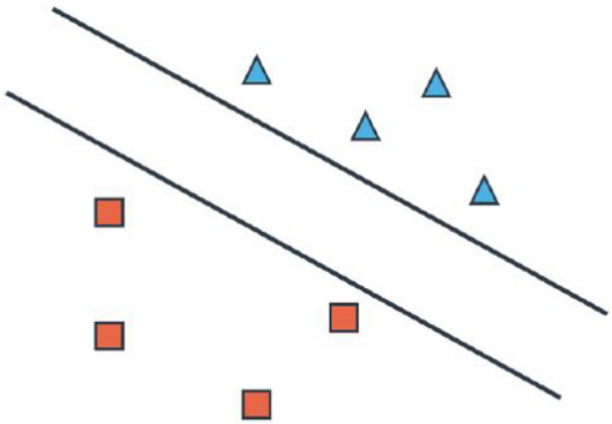
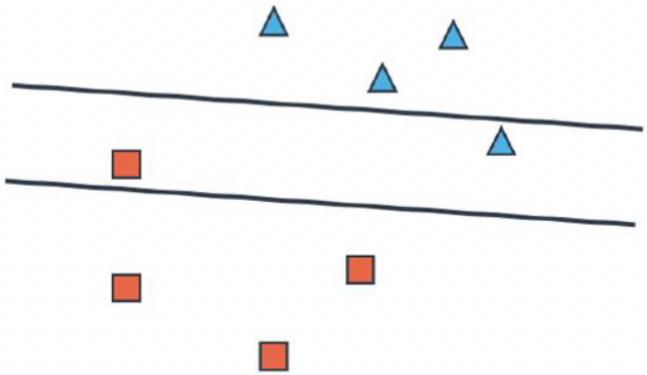
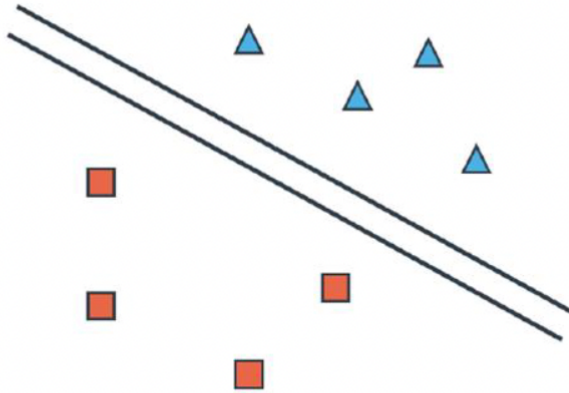


Good Classifier



Bad classifier
(makes errors)



Bad classifier
(lines are too close together)

What is more important, classification loss or distance loss?

Overall Loss Function (C Parameter)

The parameter C is used to balance the classification loss and the margin loss.

$$L(y, y') = C \cdot L_{\text{classification}} + L_{\text{margin}}$$

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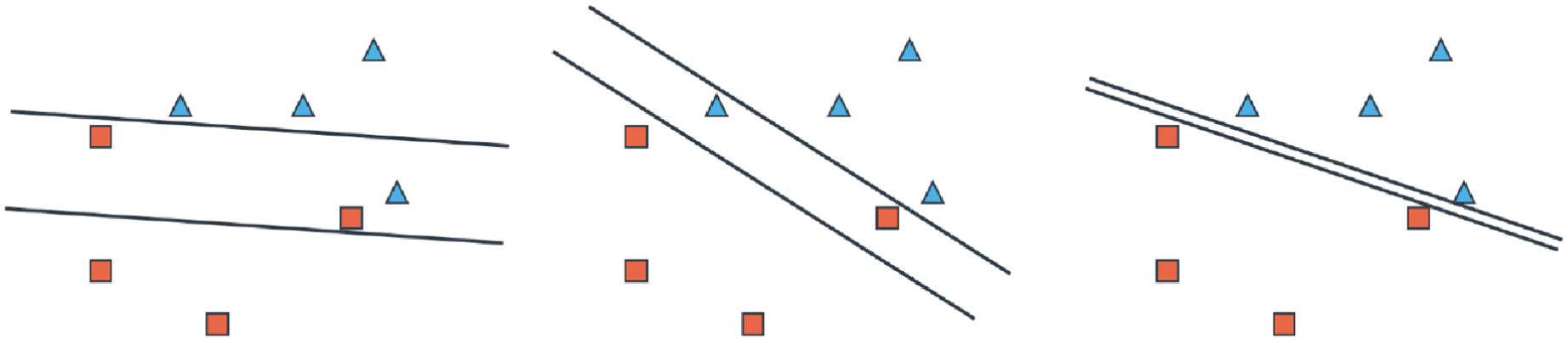

C = 0.01 C = 1 C = 100

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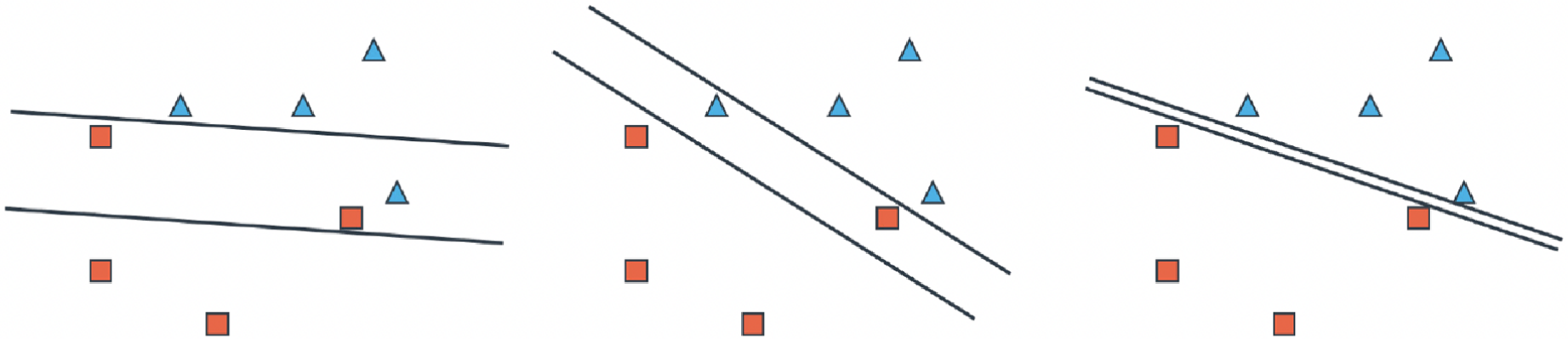

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How C parameter affects overfitting?

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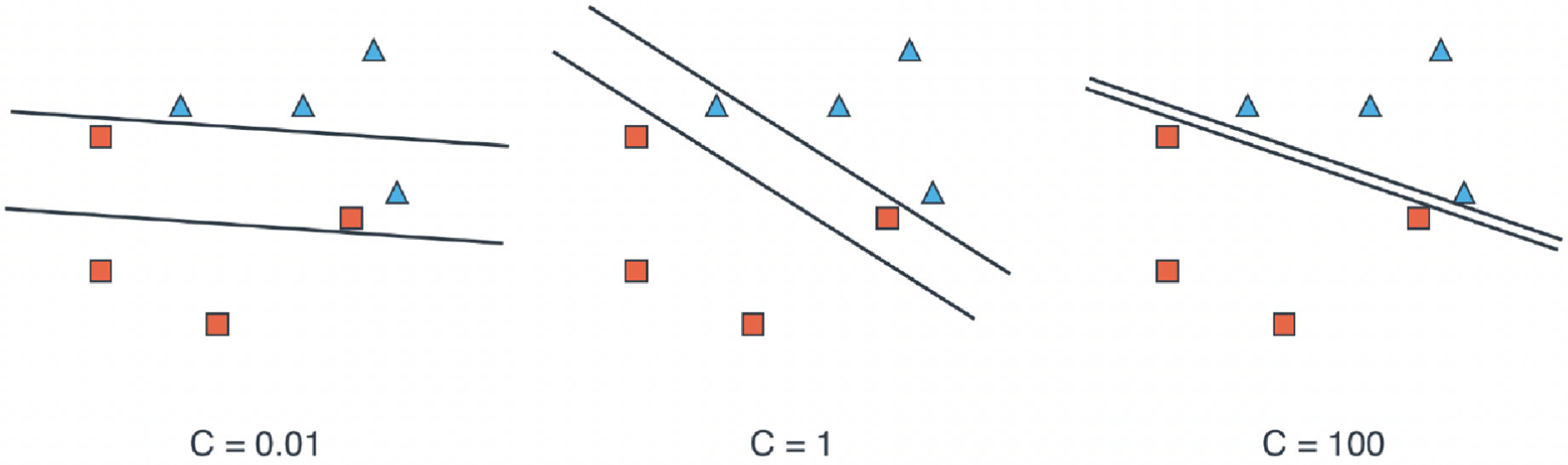


How C parameter affects overfitting?

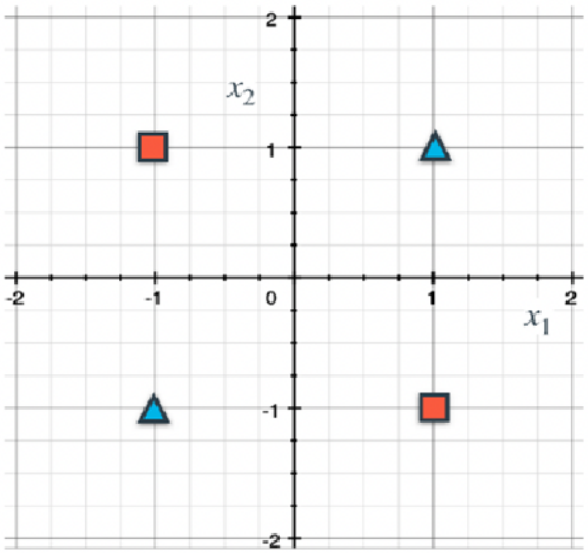
- Small C: More regularization (lower weight vector) ⇒ Less overfitting.
- Large C: Less regularization (higher weight vector) ⇒ More overfitting.

Outline

SVM Kernels

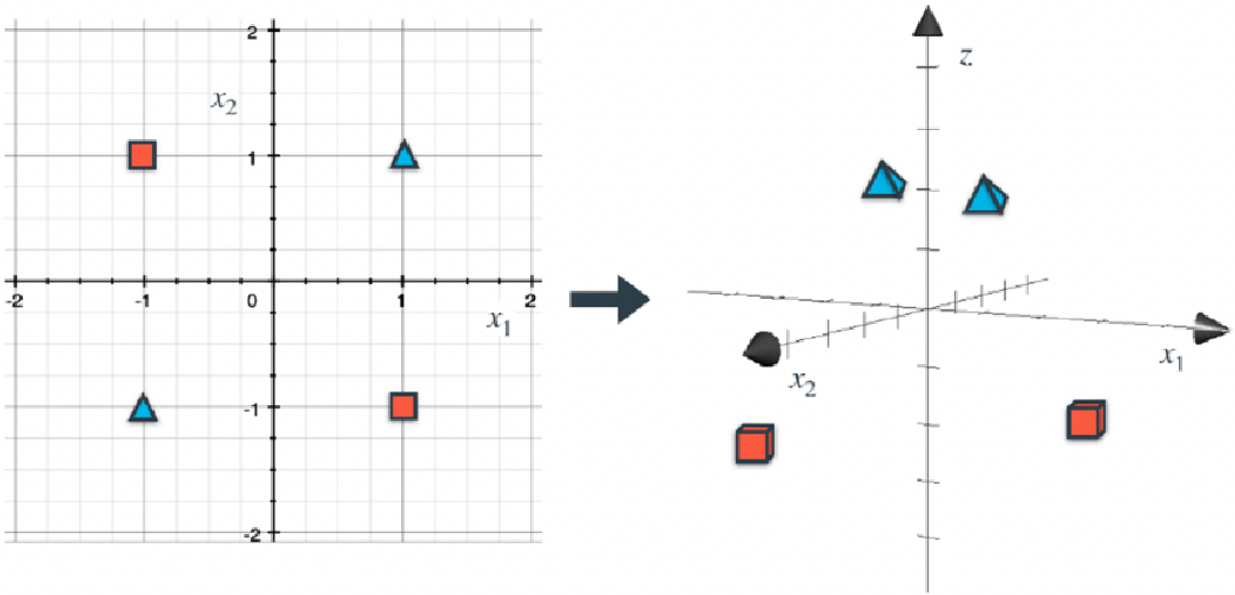
- SVM can only classify linearly separable data.
- SVM Kernels are used to transform the data into a higher dimensional space where it becomes linearly separable.
- Common Kernels:
  - Polynomial Kernel
  - RBF Kernel
  - Sigmoid Kernel

From Linear to Non-linear Boundaries



Not separable
by a line

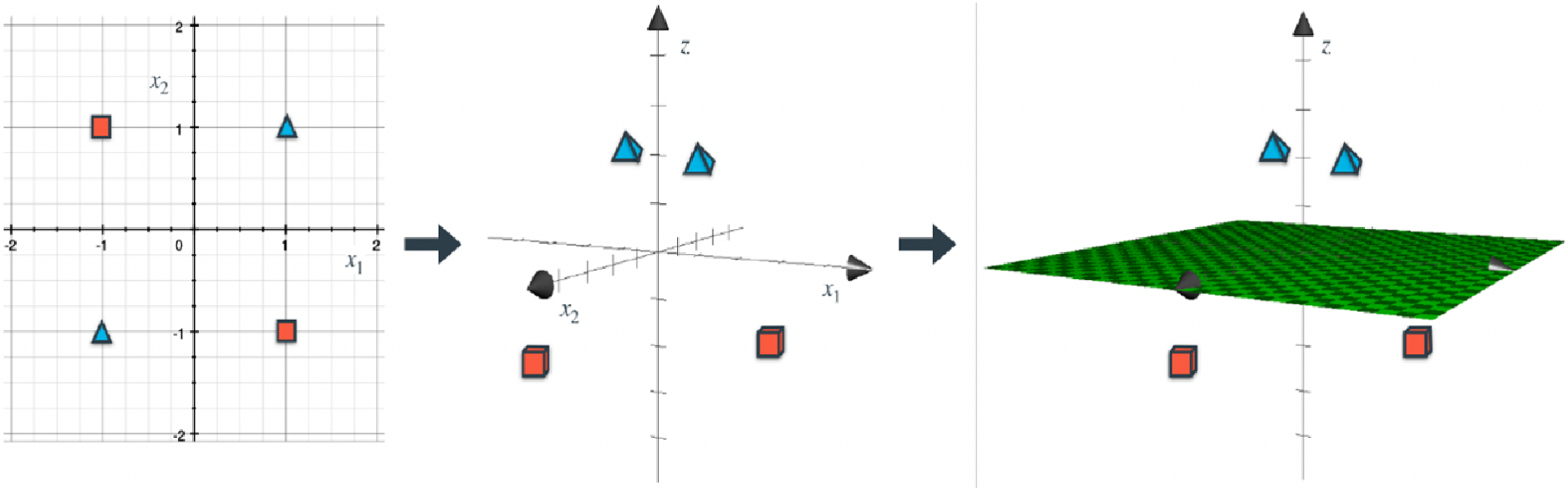
From Linear to Non-linear Boundaries



Not separable
by a line

Bring triangles up
Bring squares down

From Linear to Non-linear Boundaries



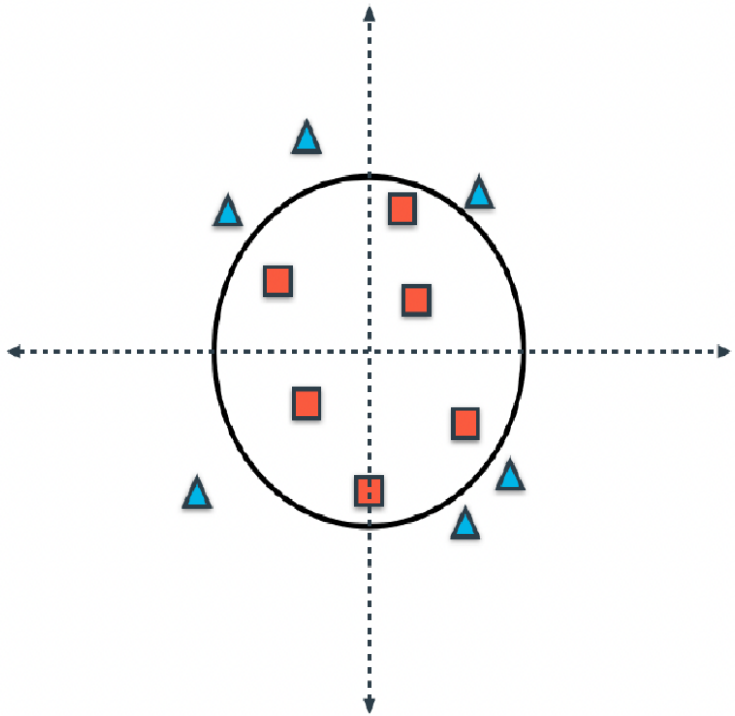
Not separable by a line

Bring triangles up
Bring squares down

Now separable by a plane

From Linear to Non-linear Boundaries

x_1	x_2	y
0.3	0.3	0
0.2	0.8	0
-0.6	0.4	0
0.6	-0.4	0
-0.4	-0.3	0
0	-0.8	0
-0.4	1.2	1
0.9	-0.7	1
-1.1	-0.8	1
0.7	0.9	1
-0.9	0.8	1
0.6	-1	1



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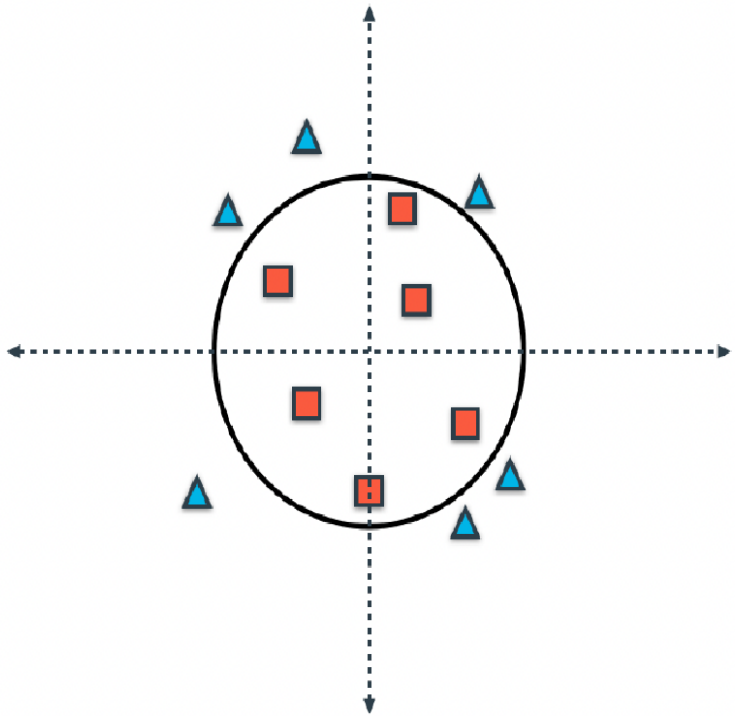

We need a circle with this formula: $x_1^2 + x_2^2 = 1$

From Linear to Non-linear Boundaries

x_1	x_2	$x_1^2 + x_2^2$	y
0.3	0.3	0.18	0
0.2	0.8	0.68	0
-0.6	0.4	0.52	0
0.6	-0.4	0.52	0
-0.4	-0.3	0.25	0
0	-0.8	0.64	0
-0.4	1.2	1.6	1
0.9	-0.7	1.3	1
-1.1	-0.8	1.85	1
0.7	0.9	1.3	1
-0.9	0.8	1.45	1
0.6	-1	1.36	1

From Linear to Non-linear Boundaries

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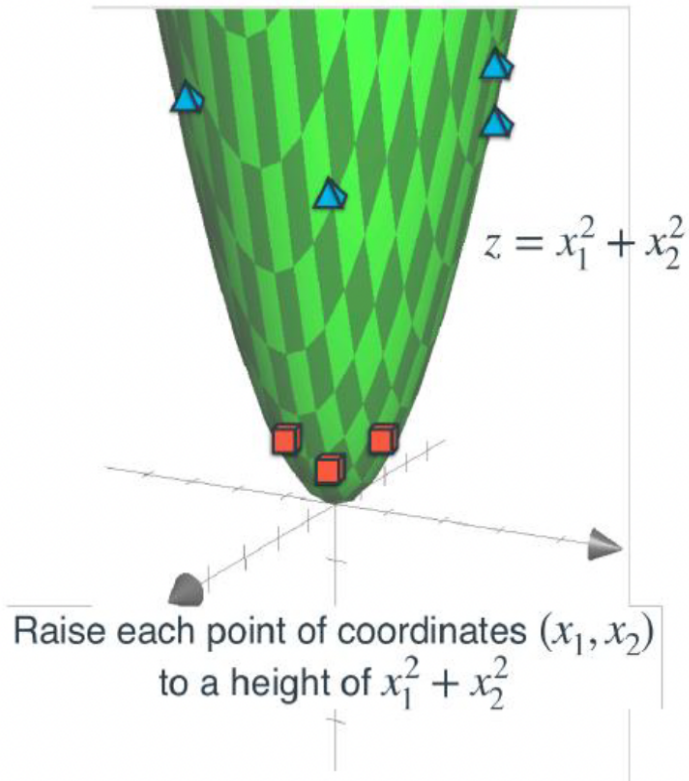
0.6	-1	1.36	1



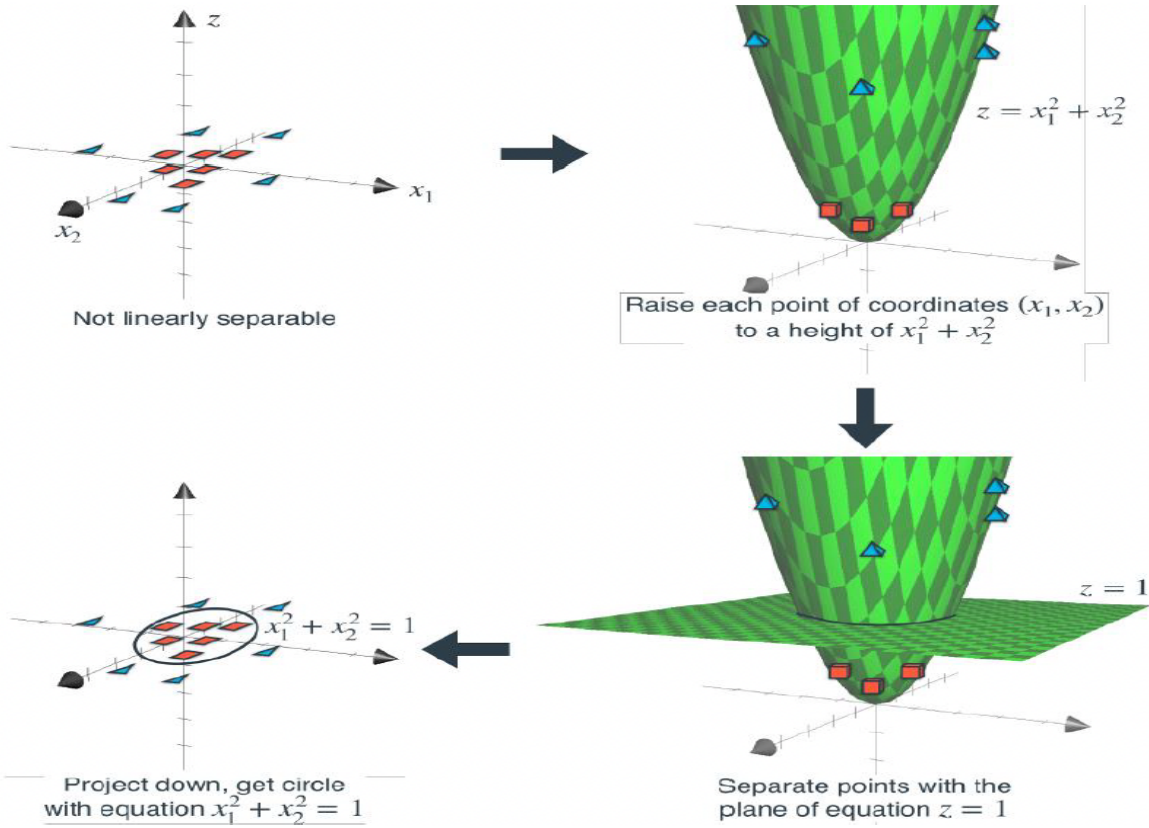
From Linear to Non-linear Boundaries

We may not have the luxury to look at a plot and eyeball an expression that will help us out.

We consider all the possible monomials of degree 2:

- These are the following three monomials:

$$x_1^2, \quad x_2^2, \quad x_1 x_2$$



Why do we need Kernels? Why not feature cross?

Suppose we have 3D input data (x_1, x_2, x_3):

- Feature cross for 2-degree:

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Suppose we have 3D input data (x_1, x_2, x_3):

- Feature cross for 2-degree:
  - Add 6 features: $x_1^2, x_2^2, x_3^2, x_1x_2, x_1x_3, x_2x_3$

Why do we need Kernels? Why not feature cross?

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- **Feature cross grows exponentially with the degree! It needs alot of memory!**
- Instead of finding higher degree features from the same sample, we can find higher degree features between different samples (using Kernels).

Polynomial Kernel

The Polynomial Kernel is defined as:

$$K(\mathbf{a}, \mathbf{b}) = (\mathbf{a} \cdot \mathbf{b} + c)^d$$

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Note: We don't need to know the explicit form of the transformed feature space. We can directly compute the kernel function on the input data on the fly.

Outline

Non-linear SVM Decision Function

The decision function for non-linear SVM is:

$$y' = f(x) = \text{sign}\left(\sum_{i=1}^{N} w_i y^{(i)} K(x^{(i)}, x) + b\right)$$

where $K(x^{(i)}, x)$ is the kernel function.

- y' is the predicted label.

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- y' is the predicted label.
- w_i are the weights we learn from the training data.
- b is the bias.
- We can think of the kernel as a **similarity measure** between the input data and all the training samples. The higher the similarity, the higher the dot product kernel value.

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- Accumulating more -ve similarity than +ve similarity will push the decision function towards -ve label.
- Cons: The weight vector length is proportional to the training samples.

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- We can use different kernels for different data.

Outline

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Hinge Loss Function and Its Derivative

Hinge Loss Function

The hinge loss function for binary classification is defined as:

$$L(y, y') = \max(0, 1 - y \cdot y')$$

where $y \in \{-1, +1\}$ is the true label and y' is the predicted score.

The derivative of the hinge loss with respect to y' is:

$$\frac{\partial L}{\partial w_i} = \begin{cases} 0, & \text{if } (1 - y \cdot y' < 0) \equiv (y \cdot y' > 1) \\ -y \cdot \frac{dy'}{dw_i}, & \text{if } (1 - y \cdot y' > 0) \equiv (y \cdot y' < 1) \end{cases}$$

Or more concisely:

$$\frac{\partial L}{\partial w_i} = -y \cdot \frac{dy'}{dw_i} \qquad if\,(y \cdot y' < 1)$$

Hinge Loss Function and Its Derivative

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Remember:

$$y' = \text{sign}\left(\sum_{i=1}^{N} w_i y^{(i)} K(x^{(i)}, x) + w_0\right)$$

Thus,

Hinge Loss Function and Its Derivative

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Remember:

$$y' = \text{sign}\left(\sum_{i=1}^{N} w_i y^{(i)} K(x^{(i)}, x) + w_0\right)$$

Thus,

$$\frac{dy'}{dw_i} = \begin{cases} y^{(i)} K(x^{(i)}, x), & \text{for } i = 1, \ldots, N \\ 1, & \text{for } i = 0 \text{ (bias term)} \end{cases}$$

SVM Classifier [Training with SGD]

- Pick random weights $w_1, w_2, \ldots, w_N$ and a random bias w_0.
- Repeat **many times**:
  1. Pick a random data point $(x^{(i)}, y^{(i)})$ where $y^{(i)} \in \{-1, +1\}$.
  2. Compute Model Prediction:

$$y'^{(i)} = \sum_{j=1}^{N} w_j y^{(j)} K(x^{(j)}, x^{(i)}) + w_0$$

  3. **Update the weights and bias if margin is violated:**

If $(y^{(i)} \cdot y'^{(i)} < 1)$:

$$w_j = w_j + \eta y^{(i)} y^{(j)} K(x^{(j)}, x^{(i)}) - \eta \lambda w_j \quad \text{for } j = 1, \ldots, N$$

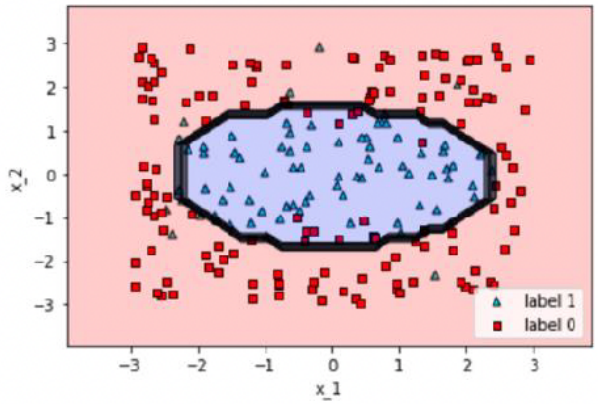
$$w_0 = w_0 + \eta y^{(i)}$$

where η is the learning rate.

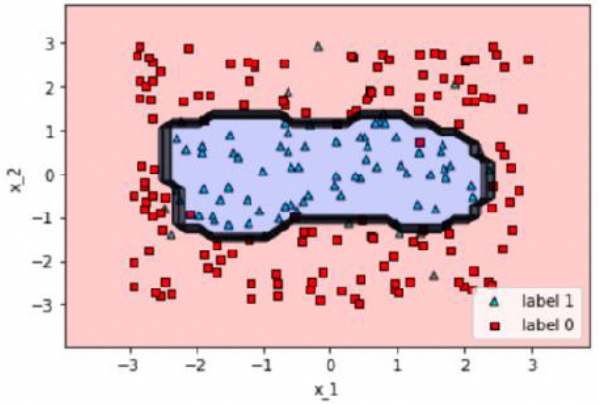
- Return the model you've obtained.

SVM Classifier [Results]

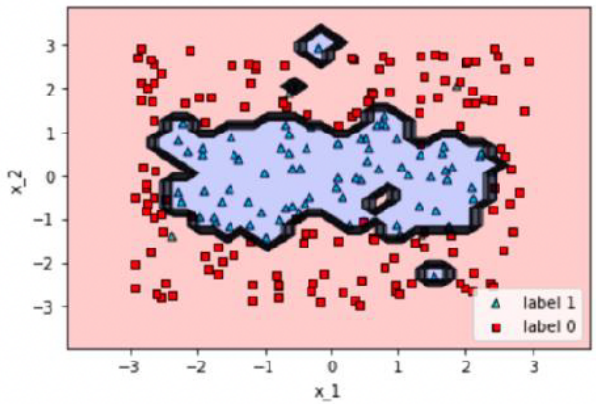
Gamma = 0.1
Accuracy: 0.8772727272727273



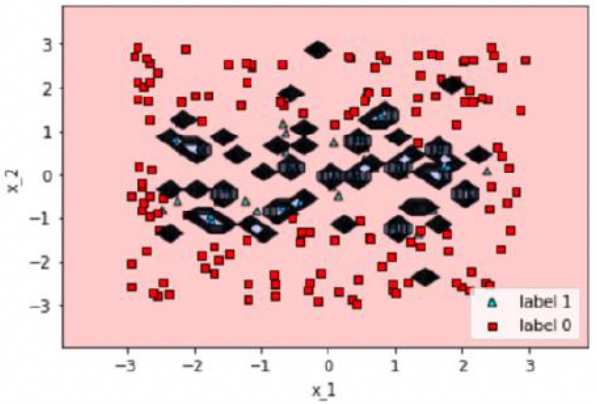
Gamma = 1
Accuracy: 0.9045454545454545



Gamma = 10
Accuracy: 0.9636363636363636



Gamma = 100
Accuracy: 0.990909090909091



Questions