ECEN 227 - Introduction to Finite Automata and Discrete Mathematics

Dr. Mahmoud Nabil
mnmahmoud@ncat.edu

North Carolina A & T State University

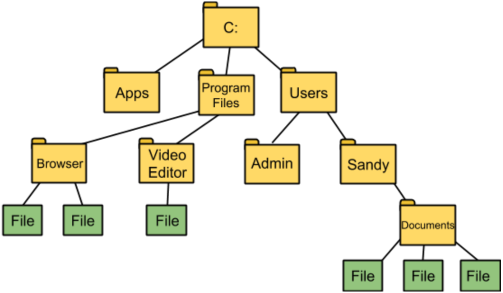
April 27, 2020

Talk Overview

1. Introduction to trees
2. Tree application examples
   - Game Tree
   - Prefix codes
3. Properties of trees
4. Tree Traversal

Tree Example

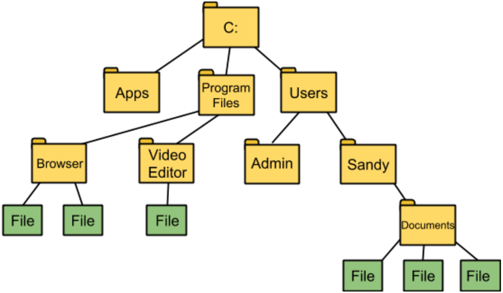
- The file system can be seen as a graph in which each folder or file is a vertex.
- There is an edge between two folders if one folder is a subfolder of the other.
- There is an edge between a file and a folder if the file resides in that folder.



Tree Defination

Tree

A tree is an undirected graph that is connected and has no cycles.



Free Tree vs Rooted Tree

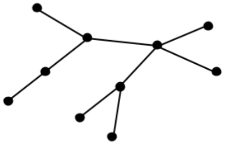
Free Tree

The tree on the left is called a free tree because there is no particular organization of the vertices and edges.

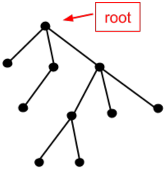
Rooted Tree

The tree on the right is called a rooted tree. The vertex at the top of the drawing is designated as the root of the tree.

Ex.



A free tree



A rooted tree

Termenologies I

Tree root

The vertex at the top of the drawing is designated as the root of the tree.

Termenologies I

Tree root

The vertex at the top of the drawing is designated as the root of the tree.

Vertex Level

The level of a vertex is its distance (i.e., number of edges) from to the root.

Termenologies I

Tree root

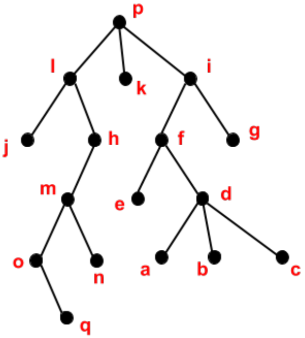
The vertex at the top of the drawing is designated as the root of the tree.

Vertex Level

The level of a vertex is its distance (i.e., number of edges) from to the root.

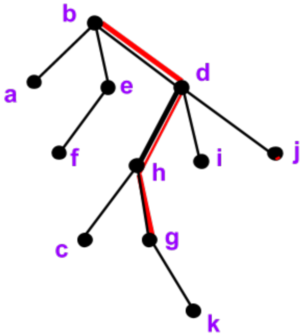
Tree Height

The height of a tree is the deepest level of any vertex.



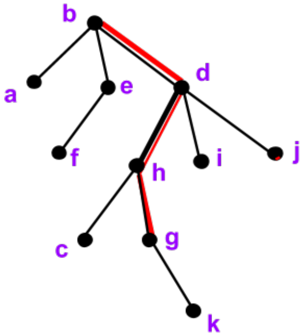
Termenologies II

- The parent of vertex v is the first vertex after v encountered along the path from v to the root.



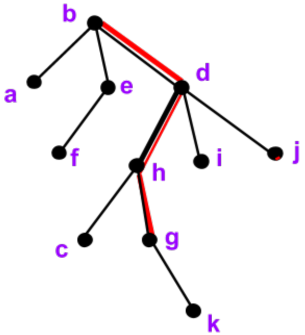
Termenologies II

- The parent of vertex v is the first vertex after v encountered along the path from v to the root.
    - (**Ex:** The parent of vertex g is h.)



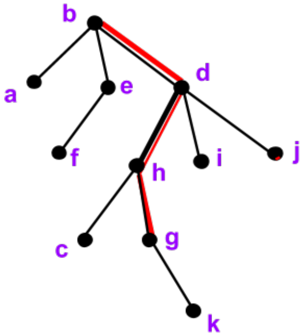
Termenologies II

- The parent of vertex v is the first vertex after v encountered along the path from v to the root.
  - (**Ex:** The parent of vertex g is h.)
- Every vertex along the path from v to the root (except for the vertex v itself) is an ancestor of vertex v.



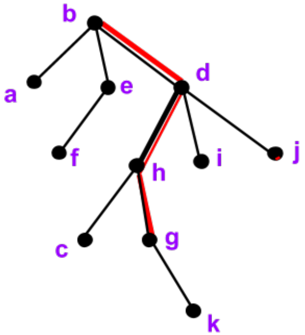
Termenologies II

- The parent of vertex v is the first vertex after v encountered along the path from v to the root.
  - (**Ex:** The parent of vertex g is h.)
- Every vertex along the path from v to the root (except for the vertex v itself) is an ancestor of vertex v.
  - (**Ex:** The ancestors of vertex g are h, d, and b.)



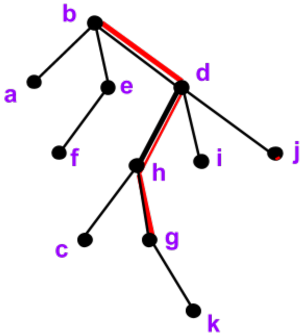
Termenologies II

- The parent of vertex v is the first vertex after v encountered along the path from v to the root.
  - (**Ex:** The parent of vertex g is h.)
- Every vertex along the path from v to the root (except for the vertex v itself) is an ancestor of vertex v.
  - (**Ex:** The ancestors of vertex g are h, d, and b.)
- If v is the parent of vertex u, then u is a child of vertex v.



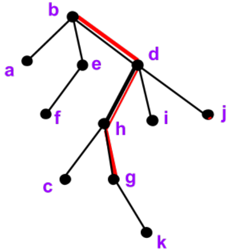
Termenologies II

- The parent of vertex v is the first vertex after v encountered along the path from v to the root.
    - (**Ex:** The parent of vertex g is h.)
- Every vertex along the path from v to the root (except for the vertex v itself) is an ancestor of vertex v.
    - (**Ex:** The ancestors of vertex g are h, d, and b.)
- If v is the parent of vertex u, then u is a child of vertex v.
    - (**Ex:** Vertices c and g are the children of vertex h.)



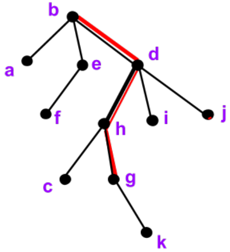
Termenologies III

- Descendants of v are children or children of children and so on.



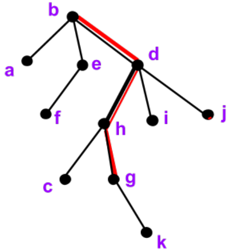
Termenologies III

- Descendants of v are children or children of children and so on.
  - (**Ex:** The descendants of vertex h are c, g, and k.)



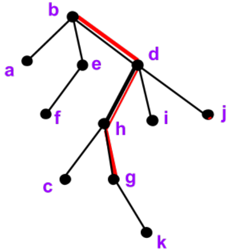
Termenologies III

- Descendants of v are children or children of children and so on.
  - (**Ex:** The descendants of vertex h are c, g, and k.)
- A leaf is a vertex with degree at most one.



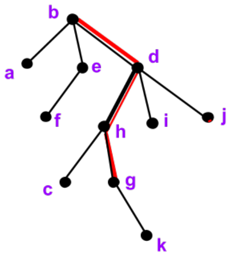
Termenologies III

- Descendants of v are children or children of children and so on.
  - (**Ex:** The descendants of vertex h are c, g, and k.)
- A leaf is a vertex with degree at most one.
  - (**Ex:** The leaves are a, f, c, k, i, and j.)



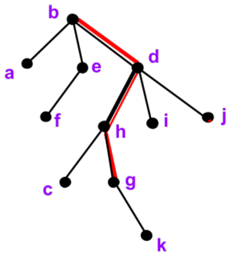
Termenologies III

- Descendants of v are children or children of children and so on.
    - (**Ex:** The descendants of vertex h are c, g, and k.)
- A leaf is a vertex with degree at most one.
    - (**Ex:** The leaves are a, f, c, k, i, and j.)
- Two vertices are siblings if they have the same parent.



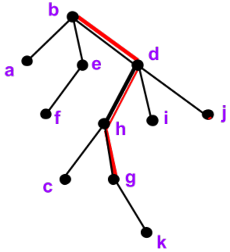
Termenologies III

- Descendants of v are children or children of children and so on.
  - (**Ex:** The descendants of vertex h are c, g, and k.)
- A leaf is a vertex with degree at most one.
  - (**Ex:** The leaves are a, f, c, k, i, and j.)
- Two vertices are siblings if they have the same parent.
  - (**Ex:** Vertices h, i, and j are siblings because they have the same parent, which is vertex d.)



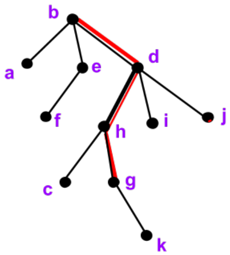
Termenologies III

- Descendants of v are children or children of children and so on.
  - (**Ex:** The descendants of vertex h are c, g, and k.)
- A leaf is a vertex with degree at most one.
  - (**Ex:** The leaves are a, f, c, k, i, and j.)
- Two vertices are siblings if they have the same parent.
  - (**Ex:** Vertices h, i, and j are siblings because they have the same parent, which is vertex d.)
- A subtree rooted at vertex v is the tree consisting of v and all v's descendants.



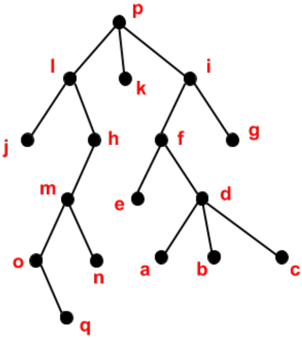
Termenologies III

- Descendants of v are children or children of children and so on.
  - (**Ex:** The descendants of vertex h are c, g, and k.)
- A leaf is a vertex with degree at most one.
  - (**Ex:** The leaves are a, f, c, k, i, and j.)
- Two vertices are siblings if they have the same parent.
  - (**Ex:** Vertices h, i, and j are siblings because they have the same parent, which is vertex d.)
- A subtree rooted at vertex v is the tree consisting of v and all v's descendants.
  - (**Ex:** The subtree rooted at h includes h, c, g, and k and the edges between them.)



Excercise

- Which vertices are ancestors of vertex n?
- Which vertices are the descendants of vertex i?
- List the leaves in the tree.
- What is the level of vertex d?
- What is the height of the tree?
- List the level four vertices.
- Draw the subtree rooted at vertex h.
- What are the siblings of vertex i?



Outline

Applications for Trees

Two applications for the trees will be covered:

- Game trees.
- Prefix codes.

Applications for Trees

Two applications for the trees will be covered:

- Game trees.
- Prefix codes.

We only highlight the data structure used by these applications. The algorithms/methods used are out of the scope of the course.

Outline

Game trees

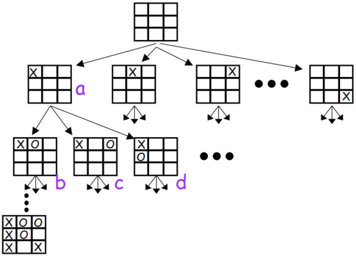
A game tree is a strategy that examines all possible moves of game, and its results, in an attempt to ascertain the optimal move.

2

Game trees

A game tree is a strategy that examines all possible moves of game, and its results, in an attempt to ascertain the optimal move.

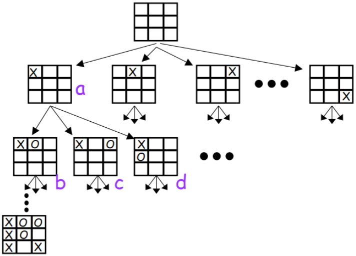
- Players alternate moves as in Tic-tac-toe.



Game trees

A game tree is a strategy that examines all possible moves of game, and its results, in an attempt to ascertain the optimal move.

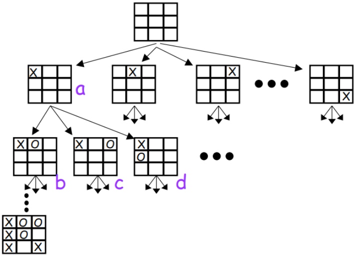
- Players alternate moves as in Tic-tac-toe.
- Artifitially intelligent player evaluate the game tree to have a best move for each game state.



Game trees

A game tree is a strategy that examines all possible moves of game, and its results, in an attempt to ascertain the optimal move.

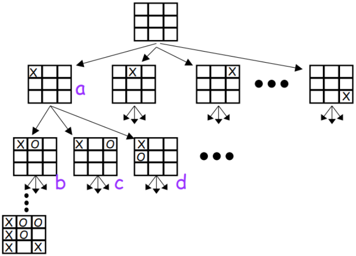
- Players alternate moves as in Tic-tac-toe.
- Artifitially intelligent player evaluate the game tree to have a best move for each game state.
- Leafs are either wins or loss situations.



Game trees

A game tree is a strategy that examines all possible moves of game, and its results, in an attempt to ascertain the optimal move.

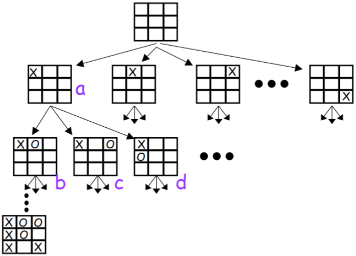
- Players alternate moves as in Tic-tac-toe.
- Artifitially intelligent player evaluate the game tree to have a best move for each game state.
- Leafs are either wins or loss situations.
- The game tree can get extremely large in complex games such as Chess.



Game trees

A game tree is a strategy that examines all possible moves of game, and its results, in an attempt to ascertain the optimal move.

- Players alternate moves as in Tic-tac-toe.
- Artifitially intelligent player evaluate the game tree to have a best move for each game state.
- Leafs are either wins or loss situations.
- The game tree can get extremely large in complex games such as Chess.
- In complex games, partial tree evaluation may be possible to overcome storage issues.



Outline

Prefix codes

- Text files are stored on computers as binary strings in which each character is assigned a binary code representing that character.
- The standard way to represent text is to have a fixed length code for every symbol in the file.
- ASCII and Unicode are examples of fixed length encodings (8 bits per character).

Motivation

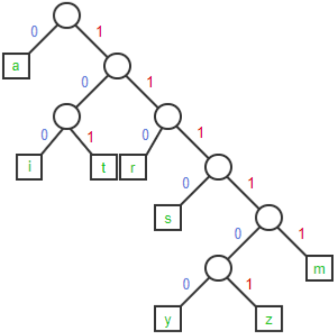
- Some characters are more frequent than the others.
- More space-efficient encodings can be achieved by variable length codes
- Algorithms for generating variable length codes are out of the scope.

Variable Length Codes

Trees are a convenient way to represent variable length codes for translating between text and binary.

Ex.

VLC Tree



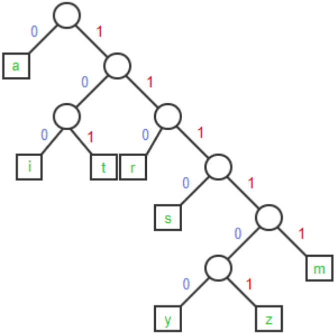
- Encode : mist

Variable Length Codes

Trees are a convenient way to represent variable length codes for translating between text and binary.

Ex.

VLC Tree



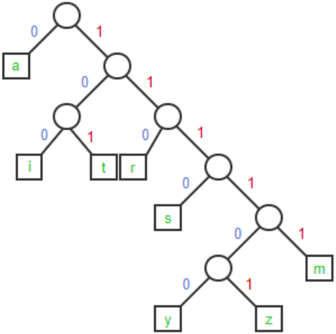
- Encode : mist
  - 111111001110101
- Decode : 11101010110

Variable Length Codes

Trees are a convenient way to represent variable length codes for translating between text and binary.

Ex.

VLC Tree



- Encode : mist
  - 111111001110101
- Decode : 11101010110
  - star

Why called prefix code? (Advantage 1)

- A prefix code has the property that the code for one character **can not be a prefix** of the code for another character.

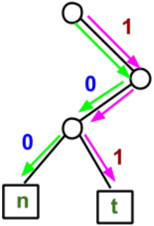
s: 101
t: 101001

s is a *prefix* of t

r: 1011
t: 101001

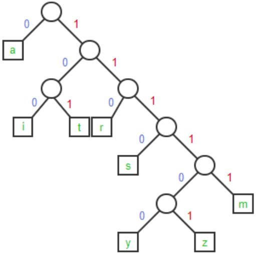
r is not a *prefix* of t

- The fact that the codes are organized as a tree in which characters are only stored at the leaves ensures the prefix property.



Paths to different leaves must eventually diverge.

Frequent characters have short codes (Advantage 2)



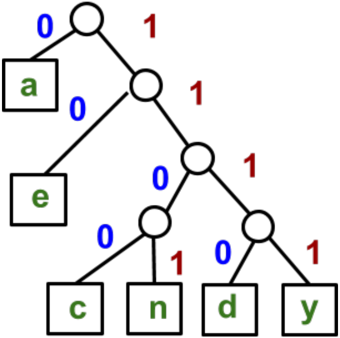
Notice that "a" is encoded using one bit (0), whereas "z" is encoded using 6 bits (111101).

Suppose a text message consists of 9,000 a's and 1,000 z's. If each letter were encoded with 8 bits, a total of 10,000*8 = 80,000 bits would be needed. But using the variable length code, only 9,000*1 + 1,000*6 or 15,000 bits would be needed.

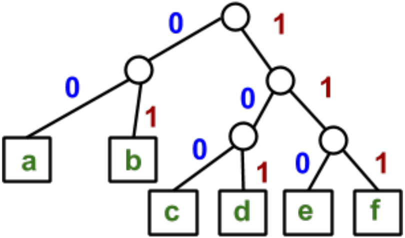
Excercise 1

- Use the tree to encode "day".
- Use the tree to encode "candy".
- Use the tree to decode "1110101101".
- Use the tree to decode "111001101110010".

VLC Tree



Excercise 2



A text file contains only characters from the set {a, b, c, d, e, f}. The frequency of each letter in the file is:

a: 5% b: 5% c: 10% d: 15% e: 25% f: 40%

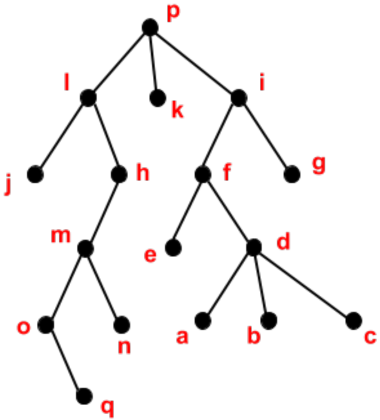
- What is the average number of bits per character used in encoding the file?
- Is there a prefix tree for the set {a, b, c, d, e, f} that would result in fewer bits per character on average for the given frequencies of the characters?

Outline

Unique paths in trees.

Theorem

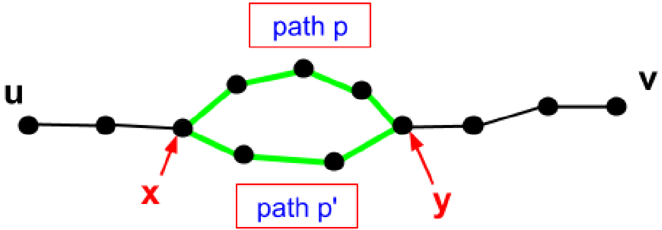
There is a unique path between every pair of vertices in a tree.



Unique paths in trees.

Theorem

There is a unique path between every pair of vertices in a tree.



- For pair of vertices u and v such that there are two distinct paths between u and v, the graph must have a cycle.
- Not a tree.

Number of leaves in a tree.

Theorem

Any free tree with at least two vertices has at least two leaves.

- Recall, the leaf is a vertex with no children. In other words, A leaf is a vertex of degree at most 1.

Excercise

- If T is a tree with n vertices, what is the most leaves that it can have?
- Draw a tree with eight vertices that has the most number of leaves possible.

Number of edges in a tree.

Theorem

Let T be a tree with n vertices and m edges, then $m = n$ - 1.

- Can be proved by induction. (Not required)

Excercise

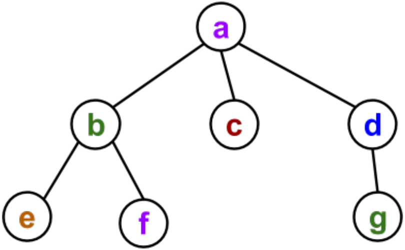
Draw a tree with the given set of properties or explain why no such tree can exist.

- Tree, seven vertices, total degree $= 14$.
- Tree, four internal vertices, four leaves.
- Tree, all vertices have degree 2.

Outline

Tree Traversal

It is a way to process the information stored in the vertices by systematically visiting each vertex in a particular order.



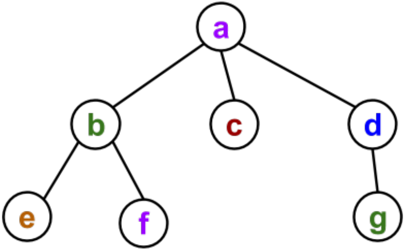
Two famous methods used in computer science for tree traversal

- Pre-order
- Post-order

Pre-order

list the root value, then "traverse" the children sub-trees from left to right in (Pre-order).

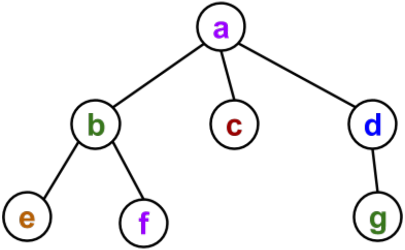
Ex.



Pre-order

list the root value, then "traverse" the children sub-trees from left to right in (Pre-order).

Ex.

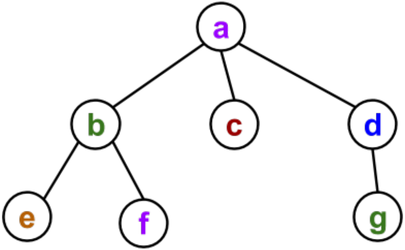


a, b, e, f, c, d, g

Post-order

"traverse" the children sub-trees from left to right in (Post-order) , then list the root value.

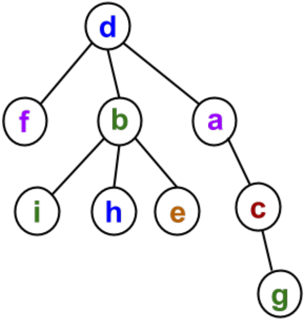
Ex.



e, f, b, c, g, d, a

Excercise

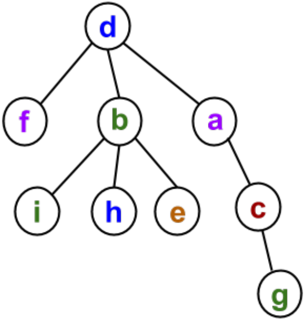
For the tree below:



- Give the order in which the vertices of the tree are visited in a post-order traversal.

Excercise

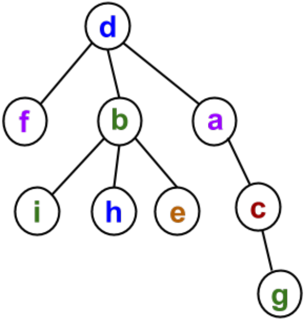
For the tree below:



- Give the order in which the vertices of the tree are visited in a post-order traversal.
  - f, i, h, e, b, g, c, a, d
- Give the order in which the vertices of the tree are visited in a pre-order traversal.

Excercise

For the tree below:



- Give the order in which the vertices of the tree are visited in a post-order traversal.
  - f, i, h, e, b, g, c, a, d
- Give the order in which the vertices of the tree are visited in a pre-order traversal.
  - d, f, b, i, h, e, a, c, g

Questions