# ECEN 227 - Introduction to Finite Automata and Discrete Mathematics

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# Talk Overview

# Outline

# Introduction

### Set

A set is a collection of objects.

### Elements

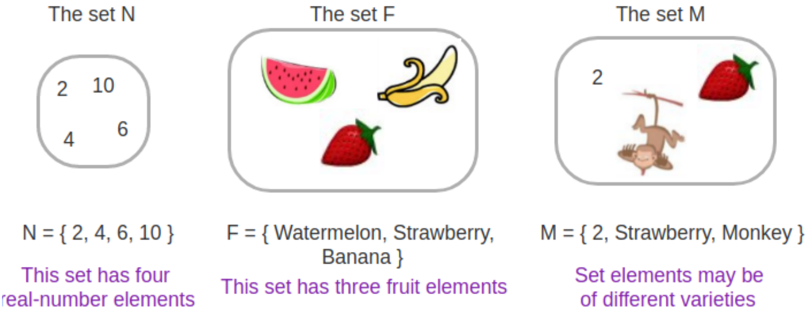
The objects in a set are called elements.

**Ex.**

$$A = \{1, 5, 3, 9\}$$

- We call the previous statement as roster notation.

# Introduction



# Notes

- The order in which the elements are listed is unimportant. So the set A can also be expressed as:

$$A = \{10, 6, 4, 2\} = \{6,4,2,10\}$$

- Repeating an element does not change the set. So the set A can also be expressed as:

$$A = \{ 2, 2, 4, 6, 10\}$$

# Empty and Null Sets

### Empty set

The set with no elements is called the empty set and is denoted by the symbol $\phi$.

### Null set

The empty set is sometimes referred to as the null set and can also be denoted by $\{\}$.

**Ex.**

- $A = \{\}$
- $B = \phi$

# Finite and Infinite Sets

**Finite set**

A finite set has a finite number of elements.

**Infinite set**

An infinite set has an infinite number of elements.

**Ex.**

- $B = \{1, 3, 5, \ldots, 99\}$ finite set
- $C = \{3, 6, 9, 12, ....\}$ infinite set

# Set Cardinality

## Set Cardinality

The cardinality of a finite set A, denoted by $|A|$, is the number of elements in A.

**Ex.**

- $A = \{1, 3, 5, 9\}$ $\quad |A| = 4$
- $B = \{1, 3, 5, \ldots, 99\}$ $\quad |B| = 50$

# Belonging

- The symbol $\in$ is used to indicate that an element is in a set.
- The symbol $\notin$ indicates that an element is not in a set.

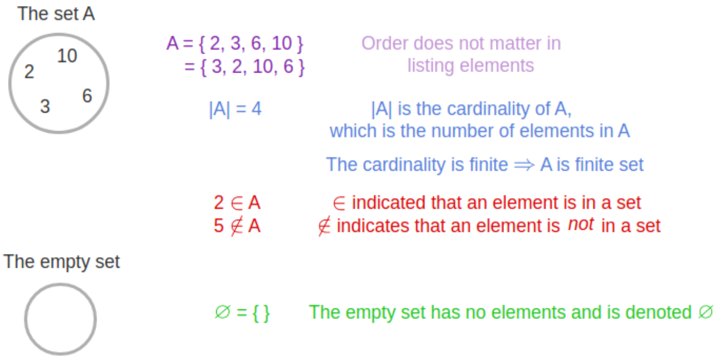
**Ex.**

$$A = \{1, 4, 7\}$$

- $1 \in A$
- $2 \notin A$

Note that, capital letters will be used as variables denoting sets, and lower case letters will be used for elements in the set.

# Example

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# Mathematical Sets

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  - Q $=\{0, 1/2, 1/3, 4/7, ...\}$
- **R: is the set of real numbers**
  - Q $=\{0, 1/2, \pi, -5/3, 2.6, \sqrt{2}, ...\}$

# Excercise

- $-3 \in Z^{+}$

# Excercise

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# Excercise

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  - False
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# Venn Diagram

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A Venn diagram is a drawing illustration of the relationships between and among sets.

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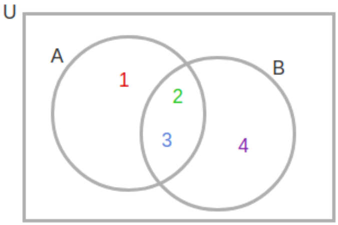


$A = \{1, 2, 3\}$

$1 \in A$ $\quad$ $4 \notin A$

$2 \in A$

$3 \in A$

$B = \{2, 3, 4\}$

## Note That

The universal set, usually denoted by the variable U, is a set that contains all elements in Venn Diagram.

# Set Builder Notation

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**Ex.**

$$C = \{x \in Z : \quad 0 < x < 100 \text{ and x is prime}\}$$

- The colon symbol ":" is read "such that".
- The description for C above would read:

"C includes all x in integers such that $0 < x < 100$ and x is prime".

# Subset and Proper Subset

**Subset $B \subseteq A$**

If every element in B is also an element of A, then B is a subset of A, denoted as $B \subseteq A$.

### Example

- $A = \{1, 2, 3, 4\}$, $B_1$=$\{1,2,3\}$, $B_2$=$\{1,2,3,4\}$, $B_3 = \{1, 2, 3, 4, 5\}$

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- $A = \{1, 2, 3, 4\}$, $B_1=\{1,2,3\}$, $B_2=\{1,2,3,4\}$, $B_3 = \{1, 2, 3, 4, 5\}$
- Does $B_1 \subseteq A$ ?

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- Does $B_1 \subseteq A$ ?
- Does $B_2 \subseteq A$ ?

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**Example**

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- Does $B_1 \subseteq A$ ?
- Does $B_2 \subseteq A$ ?
- Does $B_3 \subseteq A$ ?

# Subset and Proper Subset

**Proper Subset $B \subset A$**

If $B \subseteq A$ and there is an element of A that is not an element of B (i.e., $B \neq A$), then B is a proper subset of A, denoted as $B \subset A$.

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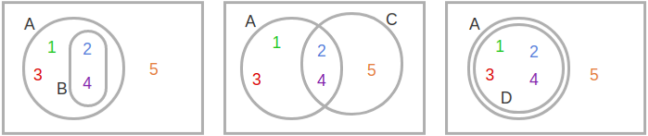
# Subset and Proper Subset




$A = \{1, 2, 3, 4\}$

$B = \{2, 4\}$

$B \subseteq A$

$3 \in A \qquad 3 \notin B$

$B \subset A$

$A = \{1, 2, 3, 4\}$

$C = \{2, 4, 5\}$

$5 \in C \qquad 5 \notin A$

$C \nsubseteq A$

$A = \{1, 2, 3, 4\}$

$D = \{1, 2, 3, 4\}$

$A \subseteq D, \;\; D \subseteq A \Rightarrow A = D$

# Outline

# Set of sets

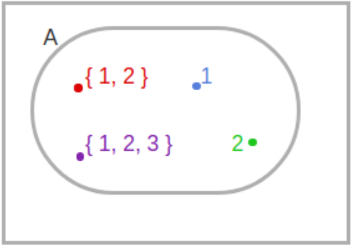
- It is possible that the elements of a set are themselves sets.

**Ex.**

$$A = \{\{1,2\}, \phi, \{1,2,3\}, \{1\}\}$$

What are the elements in A and what is $|A|$?

# Set of Sets




$A = \{\{1, 2\}, 1, 2, \{1, 2, 3\}\}$

$\{1, 2\} \in A$

$1 \in A$

$2 \in A$

$\{1, 2, 3\} \in A$

$|A| = 4$

The cardinality of set A = { {1, 2}, 1, 2, {1, 2, 3} } is 4. The elements are {1, 2}, 1, 2, and {1, 2, 3}.

# Excercise

Consider the set A:

$$A = \{\{1, 2\}, \phi, \{1, 2, 3\}, \{1\}\}$$

Mark as True or False

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**Ex.**

A = { ○, □, △ }
List all subsets:

size 0 { ∅ ,

size 1 {○} , {□} , {△ } ,

size 2 {○ , □} , {○ , △ } , {□ , △ } ,

size 3 {○ , □ , △ } } = P(A) (power set of A)

P(A) = { ∅ , {○ } , {□} , {△ } , {○ , □} , {○ , △ } , {□ , △ } , {○ , □ , △ } }

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P(A) = { ∅, {○}, {□}, {△}, {○, □}, {○, △}, {□, △}, {○, □, △} }

Can you guess the cardanality of the power set for a set of size n?

# Cardinality of Power Set

**Theorem**

*Let $A$ be a finite set of cardinality $n$. Then the cardinality of the power set of $A$ is $2^n$, or $|P(A)| = 2^n$.*

**Ex.** What is the cardinality of P({1, 2, 3, 4, 5, 6})?

# Excercise

Sets E through H are defined as follows.

- $E = \{x \in Z\colon\ x \text{ is odd}\}$
- $F = \{x \in Z^{+}\colon\ x \leq 7\}$
- $G = \{x \in Z\colon\ x < 7\}$
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Indicate whether each statement is true or false.

**Ex.**

- $G \subseteq H$

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**Ex.**

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- $\{\{0\}\} \subseteq P(G)$

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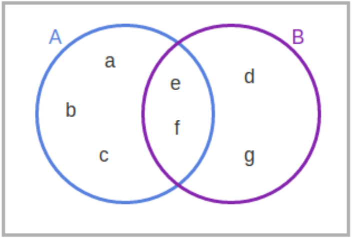
# Outline

# Intersetion Operation

- The intersection of A and B, denoted $A \cap B$ and read "A intersect B",
- It is the set of elements that are elements of both A and B.




A = { a, b, c, e, f }

B = { d, e, f, g }

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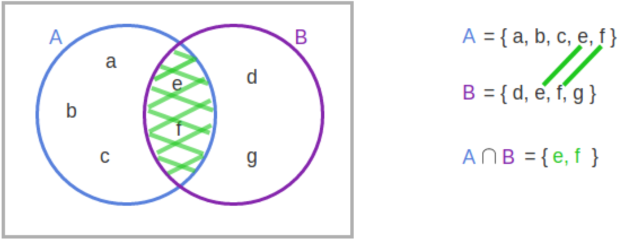

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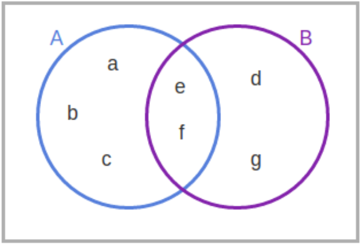


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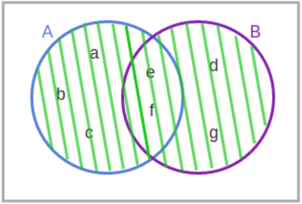


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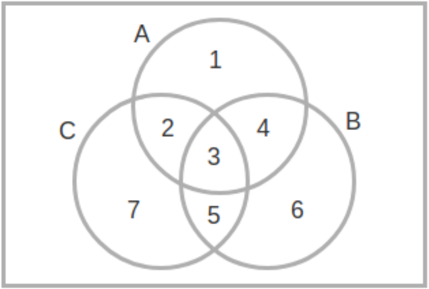
$A \cup B$ = { a, b, c, e, f, d, g }

# Excercise 1

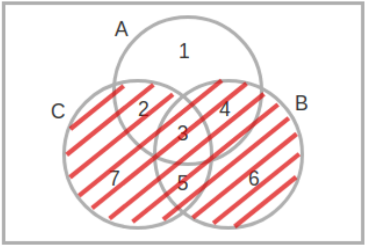



A = { 1, 2, 3, 4 }

B = { 3 , 4 , 5 , 6 }

C = { 2 , 3 , 5 , 7 }

# Excercise 1




$A = \{1, 2, 3, 4\}$

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$A \cap \underbrace{(B \cup C)}$

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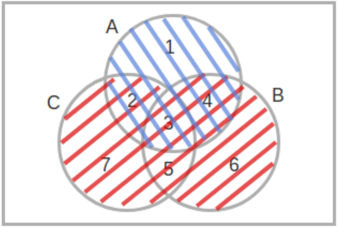


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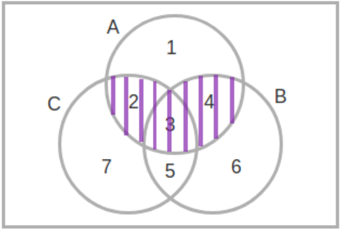


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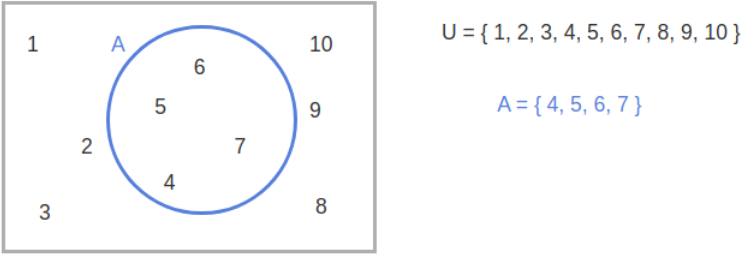
# Outline

# Set Complement

- The complement of a set A, denoted $\overline{A}$, is the set of all elements in U that are not elements of A.
- An alternative definition of $\overline{A}$ is U - A.

**Ex.**




The universal set U is {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. The set A is {4, 5, 6, 7}.

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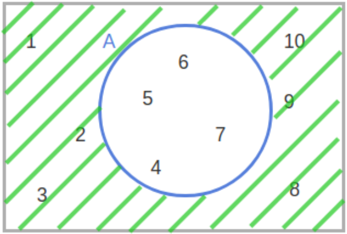


$U = \{1, 2, 3, \cancel{4}, \cancel{5}, \cancel{6}, \cancel{7}, 8, 9, 10\}$

$A = \{4, 5, 6, 7\}$

$\overline{A} = \{1, 2, 3, 8, 9, 10\}$

The complement of A is found by removing the elements of A from U. Therefore, the complement of A is {1, 2, 3, 8, 9, 10}.

# Outline

# Set Difference

- The difference between two sets A and B, denoted A - B, is the set of elements that are in A but not in B.

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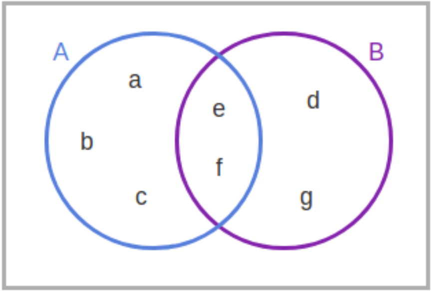


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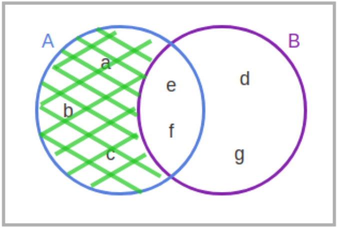



$A = \{ a, b, c, \cancel{e}, \cancel{f} \}$

$B = \{ d, e, f, g \}$

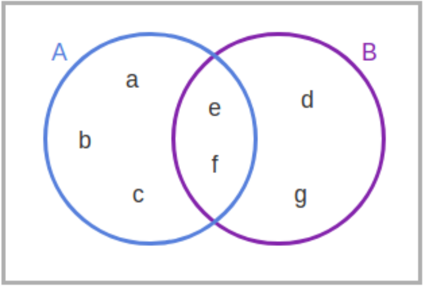
$A - B = \{ a , b , c \}$

To determine A - B, find the elements that are in both A and B (e and f) and remove those elements from A. A - B = {a, b, c}.

# Symmetric Difference

- The symmetric difference between two sets A and B, denoted $A \oplus B$, is the set of elements that are a member of exactly one of A and B but not both.

**Ex.**

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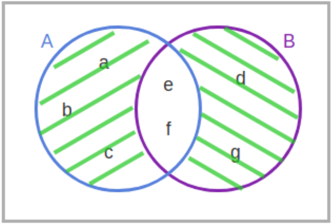



$A = \{ a, b, c, \cancel{e}, \cancel{f} \}$

$B = \{ d, \cancel{e}, \cancel{f}, g \}$

$A \oplus B = \{ a, b, c, d, g \}$

To determine $A \oplus B$, remove the elements that are in both A and B (e and f) and take the remaining elements that are in A or B. $A \oplus B = \{a, b, c, d, g\}$

# Notes on Set Difference

- The difference operation is not commutative. A - B $\neq$ B - A.
- The symmetric difference is commutative. A $\oplus$ B = B $\oplus$ A.
- An alternative definition of the set difference operation is:

$$A - B = A \cap \overline{B}$$

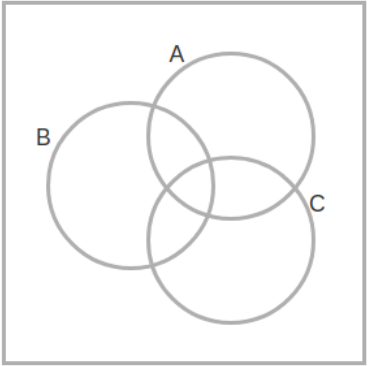
- An alternative definition of the symmetric difference operation is:

$$A \oplus B = (A - B) \cup (B - A)$$

# Operations Summary

| Operation | Notation | Description |
|---|---|---|
| Intersection | $A \cap B$ | $\{x : x \in A \text{ and } x \in B\}$ |
| Union | $A \cup B$ | $\{x : x \in A \text{ or } x \in B \text{ or both}\}$ |
| Difference | $A - B$ | $\{x : x \in A \text{ and } x \notin B\}$ |
| Symmetric difference | $A \oplus B$ | $\{x : x \in A - B \text{ or } x \in B - A\}$ |
| Complement | $\overline{A}$ | $\{x : x \notin A\}$ |

# Excercise 1




$$(\overline{A \oplus B}) \cap C$$

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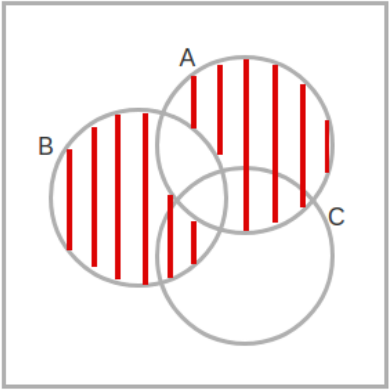


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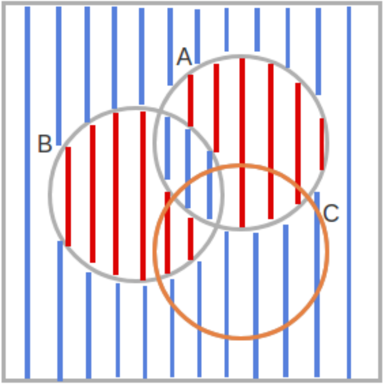


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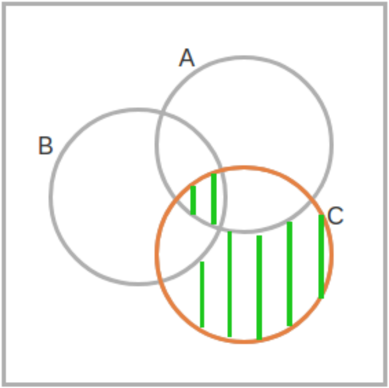


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# Outline

# Ordered Pair

### Ordered Pair

An ordered pair of elements is written $(x, y)$ where the order of elements matters.

**Notes**

- $(x, y) \neq (y, x)$ unless $x = y$.
- By contrast, $\{x, y\} = \{y, x\}$.
- An ordered list of n items is called an ordered n-tuple.

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- An ordered list of n items is called an ordered n-tuple.

  **Ex.**

  - $(w, x, y, z)$ is an ordered 4-tuple.
  - $(u, w, x, y, z)$ is an ordered 5-tuple.

# Cartesian product

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Cartesian product of A and B, denoted A x B, is the set of all ordered pairs in which the first entry is in A and the second entry is in B.

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$$\mathrm{A} \times \mathrm{B} = \{\ (\mathrm{a}, \mathrm{b}) : a \in A \text{ and } b \in B\ \}$$

**Notes**

- A x B is the same as B x A, unless A = B.
- If A and B are finite sets, then $|\mathrm{A} \times \mathrm{B}| = |\mathrm{A}| \cdot |\mathrm{B}|$

# Finite Sets Cartesian Product

B = { a, b, c }

A = { 1, 2 }

| | a | b | c |
|---|---|---|---|
| 1 | ( 1 , a ) | ( 1 , b ) | ( 1 , c ) |
| 2 | ( 2 , a ) | ( 2 , b ) | ( 2 , c ) |

A x B =
{ ( 1 , a ) , ( 1 , b ) , ( 1 , c ) }
( 2 , a ) , ( 2 , b ) , ( 2 , c )

# Finite Sets Cartesian Product

A = { 1, 2 }

B = { a, b, c }

| | 1 | 2 |
|---|---|---|
| a | ( a , 1 ) | ( a , 2 ) |
| b | ( b , 1 ) | ( b , 2 ) |
| c | ( c , 1 ) | ( c , 2 ) |

B x A

{( a , 1 ) , ( a , 2 ) ,
( b , 1 ) , ( b , 2 ) ,
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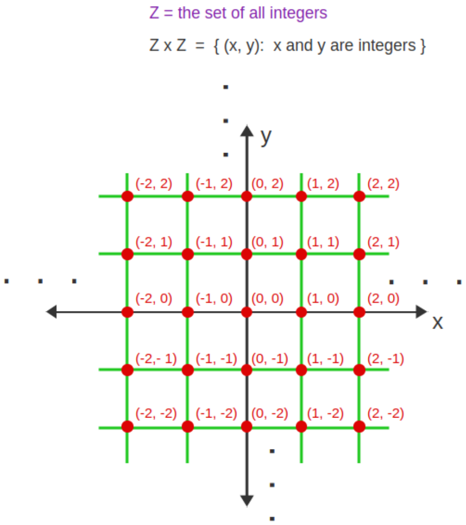
# InFinite Sets Cartesian Product




The set $Z \times Z$ forms an infinite grid of points when plotted on the x-y plane.

# Self Cartesian Product

- A × A ≡ $A^2$ or more generally:

$$A^k = \underbrace{A \times \cdots \times A}_{\text{k times}}$$

**Ex.**

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**Ex.**

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- $A^3 = \{0, 1\}^3 = \{$ (0, 0, 0), (0, 0, 1), (0, 1, 0), (0, 1, 1), (1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1) $\}$

# Strings

- If A is a set of symbols or characters, then $A^n$ can be written without parentheses and commas (i.e., called string).

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**Ex.**

- $\{0,1\}^3$ is 3-bit binary string "000" to "111".
- $\{0,1\}^n$ is n-bit binary string.

# Excercise

Given the following sets express the result as strings.

- A = {a}
- B = {b, c}
- C = {a, b, d}

**Questions**

- $A \times (B \cup C)$

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- $P(A \times B)$

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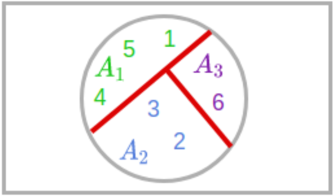
- $(A \times B) \cup (A \times C)$
  - {aa, ab, ac, ad}
- $P(A \times B)$
  - { $\phi$, {ab}, {ac}, {ab,ac} }

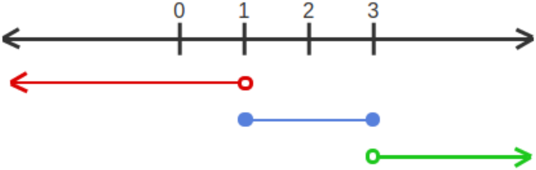
# Outline

# Partitions




$A = \{ 1, 2, 3, 4, 5, 6 \}$
$A_1 = \{ 1, 4, 5 \}$
$A_2 = \{ 2, 3 \}$
$A_3 = \{ 6 \}$

$A_1$ $A_2$ and $A_3$ form a partition of $A$




$B_1 = \{ x \in R: x < 1 \}$
$B_2 = \{ x \in R: 1 \leq x \leq 3 \}$
$B_3 = \{ x \in R: x > 3 \}$

$B_1$ $B_2$ and $B_3$ form a partition of $R$

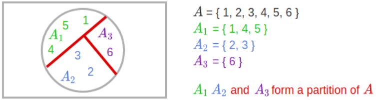
# Partitions

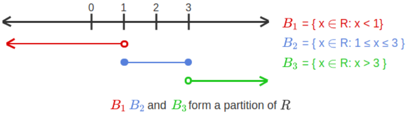
## Disjoint Sets

Two sets, A and B, are said to be disjoint if their intersection is empty ($A \cap B = \phi$).

- $A_1, A_2, \ldots, A_n$ is a partition for a non-empty set A if all of the following conditions hold:

- $A = A_1 \cup A_2 \cup \cdots \cup A_n$.
- For all i, $A_i \subseteq A$.
- For all i, $A_i \neq \phi$
- $A_1, A_2, \ldots, A_n$ are pairwise disjoint.



# Excercise

Let sets A through F be defined as follows.

- $A = \{000\}$
- $B = \{111\}$
- $C = \{0x : x \in \{0,1\}^2\}$
- $D = \{01x : x \in \{0,1\}\}$
- $E = \{1x : x \in \{0,1\}^2\}$
- $F = \{00x : x \in \{0,1\}\}$

What are the partitions of the set $\{0,1\}^3$ using one or more of the sets defined above?

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**Sol:**

- C, E
- E, D, F


Questions