ECEN 227 - Introduction to Finite Automata and Discrete Mathematics

Dr. Mahmoud Nabil
mnmahmoud@ncat.edu

North Carolina A & T State University

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Talk Overview

1. Introduction to binary relations
2. Properties of binary relations
3. Directed graphs

Outline

Relation

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A binary relation between two sets A and B is a collection of ordered pairs containing one object from each set.

Ex.

- S is the set of students at a university and C is the set of classes offered by the university.
- The relation E between S and C indicates whether a student is enrolled in a given class.
- Usually we can denote this relation as sEc.

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Note that

The relation E is subset of $S \times C$

Relations and Function

Recall that functions have more restrictions on the connection between the domain and the target as follows.

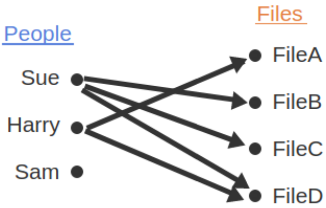
- Each element in the domain should point to one and only one element in the target.
- This is not the case in the relations

Arrow diagram for a relation

People = { Sue, Harry, Sam }

Files = { FileA, FileB, FileC, FileD }

Relation A: pAf if person p has access to file f



A = { (Sue, FileB) , (Sue, FileC) , (Sue, FileD) , (Harry, FileA) , (Harry, FileD) }

Matrix representation for a relation

People = { Sue, Harry, Sam }

Files = { File A, File B, File C, File D }

Relation A: pAf if person p has access to file f

	File A	File B	File C	File D
Sue	0	1	1	1
Harry	1	0	0	1
Sam	0	0	0	0

A = { (Sue, File B) (Sue, File C) (Sue, File D) (Harry, File A) (Harry, File D) }

Excercise

Draw the arrow diagram and the matrix representation for the following relation. Define the set A = {r, o, t, p, c} and B = {discrete, math, proof, proposition}. Define the relation $R \subseteq A \times B$ such that (letter, word) is in the relation if that letter occurs somewhere in the word.

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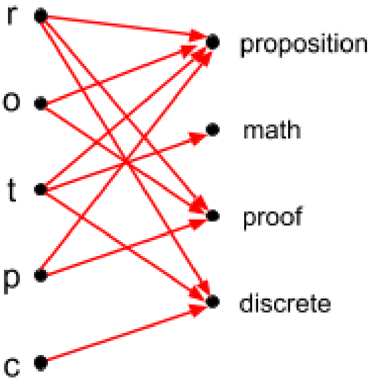

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Self Relation on a Set

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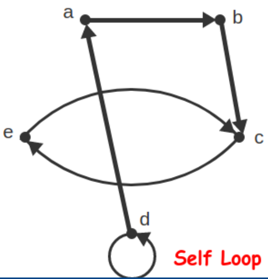
Ex.



$A = \{ a, b, c, d, e \}$

$R \subseteq A \times A$

$R = \{ (a, b) (b, c) (e, c) (c, e) (d, a) (d, d) \}$

Excercise

Draw the arrow diagram for the following relation. The domain of relation D is {2, 3, 12, 16, 27, 48}. For x, y in the domain, xDy if y is an integer multiple of x.

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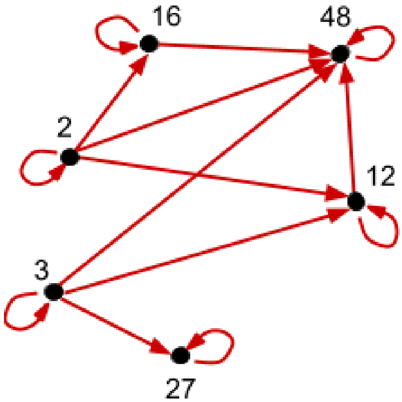
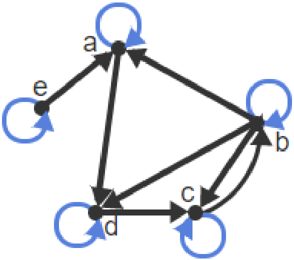


Outline

Binary relation R can be characterized by six properties. The properties are defined and illustrated using arrow diagrams.

- The relation R can be either reflexive or anti-reflexive or neither.
- The relation R can be either symmetric or anti-symmetric or neither.
- The relation R can be either transitive or not transitive.

Reflexive Relation

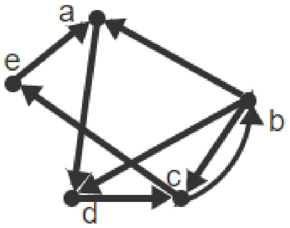


A = { a, b, c, d, e }

Relation R is reflexive if for all x ∈ A x R x

a R a, b R b, c R c, d R d, and e R e

Anti-Reflexive Relation



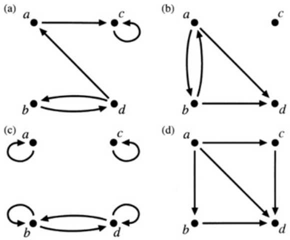
A = { a, b, c, d, e }

Relation R is
anti-reflexive
if for all x ∈ A
it is not true that
x R x

Excercise

Given the below relations indicate whether each relation is:

- reflexive, anti-reflexive, or neither



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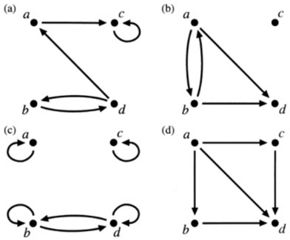
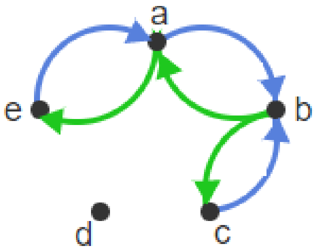


- (a) neither
- (b) anti-reflexive
- (c) reflexive
- (d) anti-reflexive

Symmetric Relation



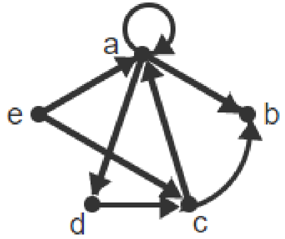
A = { a, b, c, d, e }

Relation R on A is symmetric if for all x, y ∈ A

x R y ↔ y R x

x R y ↔ y R x

Anti-Symmetric Relation



A = { a, b, c, d, e }

Relation R is anti-symmetric if for all x, y ∈ A

x R y and y R x → x = y

Note: there is no 

Excercise

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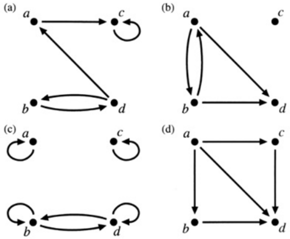

Excercise

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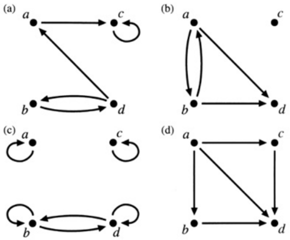
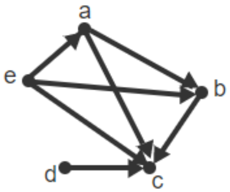
- symmetric, anti-symmetric, or neither



- (a) neither
- (b) neither
- (c) symmetric
- (d) anti-symmetric

Transitive Relation



A = { a, b, c, d, e }

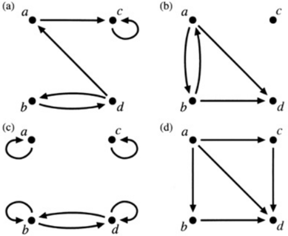
Relation R on A is transitive if for all x, y, z ∈ A if x R y and y R z, then x R z

e R a and a R b ⟶ e R b
e R b and b R c ⟶ e R c
e R a and a R c ⟶ e R c
a R b and b R c ⟶ a R c

Excercise

Given the below relations indicate whether each relation is:

- transitive or not transitive



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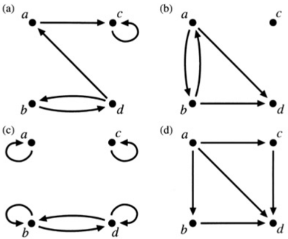

- (a) not transitive
- (b) transitive
- (c) transitive
- (d) transitive

Excercise 3

Given the below relation indicate whether the relation is:

- reflexive, anti-reflexive, or neither
- symmetric, anti-symmetric, or neither
- transitive or not transitive

The domain of the relation L is the set of all real numbers. For $x, y \in R$, xLy if $x < y$.

Excercise 3

Given the below relation indicate whether the relation is:

- reflexive, anti-reflexive, or neither
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The domain of the relation L is the set of all real numbers. For $x, y \in R$, xLy if $x < y$.

Answer.

- anti-reflexive: For any real number x, it is always false that $x < x$.
- anti-symmetric: For any two real numbers x and y, it can not be true that $x < y$ and $y < x$.
- transitive: If $x < y$ and $y < z$, then $x < z$.

Excercise 3

Given the below relation indicate whether the relation is:

- reflexive, anti-reflexive, or neither
- symmetric, anti-symmetric, or neither
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The domain of the relation L is the set of all real numbers. For $x, y \in R$, xLy if $x \leq y$.

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Answer.

- reflexive: For any real number x, it is always true that $x \le x$.
- anti-symmetric: For any two real numbers x and y, if $x \le y$ and $y \le x$, then $x = y$.
- transitive: If $x \le y$ and $y \le z$, then $x \le z$.

Outline

Graph

Graph is simply a relation over set. It is used widely in computer science topics.

Ex.

- Internet pages.
- Friends on facebook.
- Train/Bus stations.
- Communication network.
- etc.

Directed Graph (digraph)

Digraph

A directed graph (or digraph, for short) consists of a pair (V, E). V is a set of vertices, and E, a set of directed edges.

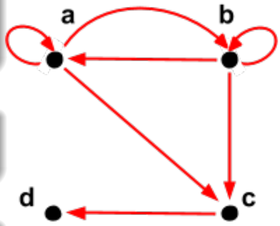
Vertex

An individual element of V is called a vertex.

Edge

A connection between two vertices.

Example

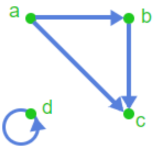


Graph Example

Graph G = (V, E)

V = { a, b, c, d }

E = { (a, b), (b, c), (a, c), (d, d) }



a is the tail of edge (a, b)
b is the head of edge (a, b)

The in-degree of c is 2
The out-degree of b is 1
The in-degree of d is 1

In-degree

The in-degree of a vertex is the number of edges pointing into it.

Out-degree

The out-degree of a vertex is the number of edges pointing out of it.

Walks and directed Graph

A walk in a directed graph G is a sequence of alternating vertices and edges that starts and ends with a vertex.

$$< v_0, v_1, v_2, ..., v_n >$$

Walk length

The length of a walk is l, the number of edges in the walk.

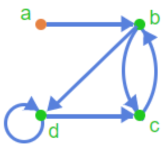
Open walk

An open walk is a walk in which the first and last vertices are different.

Closed walk

A closed walk is a walk in which the first and last vertices are the same.

Example



Walk:

⟨ a, b, c, b, d ⟩ walk length = 4

The walk is open because
the first and last vertices are not the same

Walk:

⟨ b, d, c, b, c, b ⟩ walk length = 5

Edge (c, b) occurs twice
so (c, b) is counted twice

This walk closed because the
first and last vertices are the same.

Closed walk of length 1: ⟨ d, d ⟩

Closed walk of length 0: ⟨ a ⟩

Definations

Trail

A trail is an open walk in which no edge occurs more than once.

Path

A path is a trail in which no vertex occurs more than once.

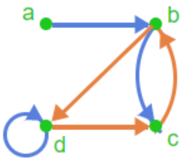
Circuit

A circuit is a closed walk in which no edge occurs more than once.

Cycle

A cycle is a circuit in which no vertex occurs more than once, except the first and last vertices which are the same.

Example



Walk:

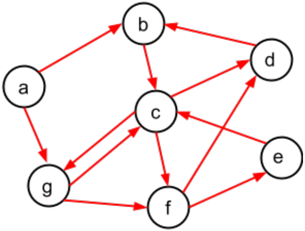
⟨ a, b, c, b, d ⟩ No edge occurs more than once. So this open walk is a trail.

b is reached twice
So this trail is not a path

⟨ b, d, c, b ⟩ No edge occurs more than once. So this closed walk is a circuit.

The circuit is a cycle because only
the first and last vertices are repeated.

Excercise



- What is the in-degree of vertex d?
- What is the out-degree of vertex c?
- What is the head of edge (b, c)?
- What is the tail of edge (g, f)?
- List all the self-loops in the graph.
- Is <a, g, f, c, d> a walk in the graph? Is it a trail? Is it a path?
- Is <a, g, f, d, b> a walk in the graph? Is it a trail? Is it a path?
- Is <c, g, f, e> a circuit in the graph? Is it a cycle?

Questions