ECEN 227 - Introduction to Finite Automata and Discrete Mathematics

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Talk Overview

1. Sequences
2. Recurrence relations
3. Summation

Outline

Sequence

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A sequence is a special type of function in which the domain is a consecutive set of integers.

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Ex.

2, 4, 6, 8, 10

Finite and Infinite Sequence

When the sequence goes on forever it is called an infinite sequence, otherwise it is a finite sequence.

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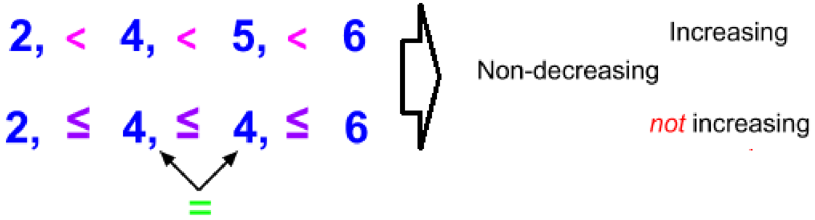
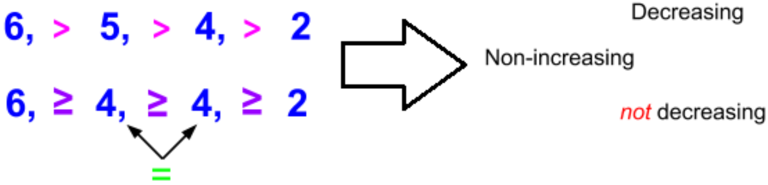
7, 6, 5, 4, 3, 2, 1 Initial index =1, Final index= 7 (Finite Sequence)

Increasing and non-decreasing sequences.



Decreasing and non-increasing sequences.



Excercise I

Indicate whether the sequence is increasing, decreasing, non-increasing, or non-decreasing. You can assume that the sequences start with an index of 1. Logs are to base 2.

- The n^{th} term is $\lceil \sqrt{n} \rceil$.

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  - 0, 1, 2, 2, 3, 3, 3,...

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Arithemtic Sequence

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An arithmetic sequence is a sequence of real numbers where each term after the initial term (a_0) is found by taking the previous term and adding a fixed number called the common difference (d).

- An arithmetic sequence can be finite or infinite.
- n^{th} term $= a_0 + \text{d} \times (\text{n})$ Assuming initial index is zero

Ex.

- Initial term $(a_0) = 1$

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Example

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- The sequence a_n is an arithmetic sequence with an initial value of 500 and a common difference of 10.
- After n weeks of collecting, $a_n = 500 + 10n$.

Geometric Sequence

Geometric Sequence

A geometric sequence is a sequence of real numbers where each term after the initial term (a_0) is found by taking the previous term and multiplying by a fixed number called the common ratio (r).

- A geometric sequence can be finite or infinite.
- n^{th} term $= a_0 \times \mathrm{r}^n$ Assuming initial index is zero

Ex.

- Initial term $(a_0) = 4$,
- Common Ratio $(\mathrm{r}) = 1/2$
- 4, 2, 1, 1/2, 1/4, ...
- n^{th} term $= 4 \times (1/2)^n$ Assuming initial index is zero

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- Since the interest rate is annual and compounded monthly, (6/12)% of the current amount is added to the account each month.
- a_0=1000 is the initial balance in the account, and a_n is the balance in the account after n months of earning interest. Each month, the balance in the account is 1.005 times the amount that was in the account in the previous month.
- The sequence a_n is a geometric sequence with $a_n = 1000 \times (1.005)^n$

Outline

Recurrence relation

Fibonacci sequence

$$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, \ldots$$

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Rule is: $a_n = a_{n-1} + a_{n-2}$ n≥2

$a_0 = 0, \; a_1 = 1$

Recurrence relation

A rule that defines a term $\mathbf{a}_n$ as a function of previous terms in the sequence is called a recurrence relation.

Excercise

Give the first six terms of the following sequences.

- $c_1 = 4$, $c_2 = 5$, and $c_n = c_{n-1} * c_{n-2}$ for $n \geq 3$.

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  - 2, 1, 5, 21, 110, 681

Recurrence Relation for Arithemtic Sequence

- a_0 = a (initial value)
- $a_n = d + a_{n-1}$ for n ≥ 1 (recurrence relation)
- Initial value = a. Common difference = d.

Ex.

- Initial term = 1
- Common Difference = 4
- 1, 5, 9, 13, ...

Recurrence Relation for Geometric Sequence

- a_0 = a (initial value)
- a_n = r × a_{n-1} for n ≥ 1 (recurrence relation)
- Initial value = a. Common ratio = r.

Ex.

- Initial term = 1
- Common ratio = 2
- 1, 2, 4, 8, 16, ...

Arithemtic vs Geometric Sequence

	Arithemtic Sequence	Geometric Sequence
n^{th} term iterative	$a_n = a_0 + d \times n$	$a_n = a_0 \times r^n$
n^{th} term recursive	$a_n = \mathrm{d} + a_{n-1}$	$a_n = \mathrm{r} \times a_{n-1}$

Example

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Answer.

$$a_0 = \$20000$$
$$a_n = (1.0025) \times a_{n-1} - 500$$

$$a_0 = \$20{,}000$$
$$a_1 = \$19{,}550$$
$$a_2 = \$19{,}099$$
$$a_3 = \$18{,}647$$
$$\dots$$

Outline

Summation

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- Consider a sequence:

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The notation to express the sum of the terms in that sequence is:

$$\sum_{i=s}^{i=t} a_i = a_s + a_{s+1} + \cdots + a_t$$

- The variable i is called the index of the summation.
- The variable s is the lower limit
- The variable t is the upper limit of the summation.

Example 1

Suppose we want to write a summation for the sequence

$$n^2 \text{ for n} = 1,\ 2, \ldots 5$$

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$$n^2 \text{ for n} = 1,\ 2, \ldots 5$$

Then:

$$\sum_{j=1}^{j=5} j^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2$$

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Evaluate the following summations.

- $\sum_{k=-1}^{k=4} k^2$

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    - $(-1)^2 + 0^2 + 1^2 + 2^2 + 3^2 + 4^2 = 31$
- $\sum_{k=0}^{k=4} 2^k$

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- $\sum_{k=0}^{k=4} 2^k$
  - $2^0 + 2^1 + 2^3 + 2^4 = 31$
- $\sum_{k=-3}^{k=2} k^3$

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- $\sum_{k=0}^{k=4} 2^k$
  - $2^0 + 2^1 + 2^3 + 2^4 = 31$
- $\sum_{k=-3}^{k=2} k^3$
  - $(-3)^3 + (-2)^3 + (-1)^3 + 0^3 + 1^3 + 2^3 = -27$

Note

It is important to use parentheses if you have more than one term.
E.g.

- $\sum\limits_{j=1}^{j=4} (j^2 + 1)$
- $\sum\limits_{j=1}^{j=4} j^2 + 1$

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E.g.

- $\sum_{j=1}^{j=4}(j^2+1)$
- $\sum_{j=1}^{j=4} j^2+1$

$$\sum_{j=1}^{j=4}(j^2+1) = (1^2+1)+(2^2+1)+(3^2+1)+(4^2+1)$$

$$\sum_{j=1}^{j=4} j^2+1 = (\sum_{j=1}^{j=4} j^2)+1 = (1^2)+(2^2)+(3^2)+(4^2)+1$$

Pulling out a final term from a summations

$$\sum_{j=m}^{j=n} a_j = \sum_{j=m}^{j=n-1} a_j + a_n$$

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Ex.

$$\sum_{j=1}^{n} (j+1)^2 = (1+1)^2 + (2+1)^2 + \ldots + ((n-1)+1)^2 + (n+1)^2$$

$$= \sum_{j=1}^{n-1} (j+1)^2 + (n+1)^2$$

Excercise

Pull out the final term from the following summations.

- $\sum_{j=0}^{j=n+2} 2^{j-1}$

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- $\sum_{k=0}^{k=m+2} (k^2 - 4k + 1)$

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- $\sum_{k=0}^{k=m+2} (k^2 - 4k + 1)$
  - $\sum_{k=0}^{k=m+1} (k^2 - 4k + 1) + (m+2)^2 - 4(m+2) + 1$

Change of variables in summations

$$\sum_{j=1}^{j=n} (j+2)^3$$

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Three steps to be taken:

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Three steps to be taken:

- Replace term in the summation.
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$$\sum_{k=3}^{k=n+2} (k)^3$$

Excercise

Pull out the final term from the following summations.

- Substitute variable j for k, where j = k - 1, in the summation
  $$\sum_{k=0}^{k=n-1} 2^{k-2}$$

Excercise

Pull out the final term from the following summations.

- Substitute variable j for k, where j = k - 1, in the summation $\sum_{k=0}^{k=n-1} 2^{k-2}$
  - $\sum_{j=-1}^{j=n-2} 2^{j-1}$
- Substitute variable k for j, where k = j - 4, in the summation $\sum_{j=4}^{j=17} (2j+4)$

Excercise

Pull out the final term from the following summations.

- Substitute variable j for k, where j = k - 1, in the summation $\sum_{k=0}^{k=n-1} 2^{k-2}$
  - $\sum_{j=-1}^{j=n-2} 2^{j-1}$
- Substitute variable k for j, where k = j - 4, in the summation $\sum_{j=4}^{j=17} (2j+4)$
  - $\sum_{k=0}^{k=13} (2k+12)$

Closed forms for sums

Closed form form a summuation

A closed form for a sum is a mathematical expression that expresses the value of the sum without summation notation.

Ex.

$$\sum_{k=1}^{k=n} k = \frac{n(n+1)}{2}$$

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- Arithemtic sequences have a closed form.
- Geometric sequences have a closed form.

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Arithemtic Sequence Summation

For any integer n ≥ 1:

$$\sum_{k=0}^{k=n-1}(a+kd) = a \times n + \frac{d(n-1)n}{2}$$

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Geometric Sequence Summation

For any real number r ≠ 1 and any integer n ≥ 1:

$$\sum_{k=0}^{k=n-1} ar^k = \frac{a(r^n-1)}{r-1}$$

Excercise 1

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- $\sum_{k=0}^{k=100} 3 \times (1.1)^k$
  - $\frac{3(1.1^{101}-1)}{0.1} \approx 454730.2072$

Excercise 2

A Silicon Valley company purchases 3 new cars at the end of every month. Let a_n denote the number of cars he has after n months. Let $a_0 = 23$.

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  - $50 \sum_{i=0}^{i=23} (23 + 3i) = 50[23 \times 24 + \frac{3 \times 23 \times 24}{2}]$

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  - $30(1.12)^{12}$
- If each rabbit consumes 10 pounds of rabbit food each year, then how much rabbit food is consumed in 10 years? For simplicity, you can omit the food consumed by the baby rabbits born in a given year. For example, suppose the farm starts tabulating rabbit food on January 1, 2012 at which time the rabbit population is 30. You will count the food consumed by those 30 rabbits during 2012. You won't count the food consumed by the rabbits born in 2012 until after January 1, 2013.

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  - $10 \sum_{i=0}^{i=9} 30(1.12)^i = 300(\frac{1.12^{10}-1}{0.12})$

Questions